

$$\overline{\langle \mathbb{Z} \rceil \mathbb{Z}_p \rangle}^k = {}^k p$$

$$\langle \mathbb{Z} \rceil \mathbb{Z}_p \rangle^n \xleftarrow[\text{code}]{\downarrow} \langle \mathbb{Z} \rceil \mathbb{Z}_p \rangle^m$$

$$\downarrow^m p = \frac{\downarrow \in {}^n \mathbb{Z}}{\downarrow + {}^n \mathbb{Z}_p \in \downarrow^m \langle \mathbb{Z} \rceil \mathbb{Z}_p \rangle}$$

$$\langle \mathbb{Z} \rceil \mathbb{Z}_p \rangle^n \rceil \overline{\langle \mathbb{Z} \rceil \mathbb{Z}_p \rangle}^m \simeq \langle \mathbb{Z} \rceil \mathbb{Z}_p \rangle^{n-m} \Rightarrow \overline{\langle \mathbb{Z} \rceil \downarrow^m p \rangle}^n = {}^{n-m} p$$