

$$\mathfrak{A}^t = \underbrace{1-t}_{\text{h}} \mathfrak{A}^0 + t \mathfrak{A}^1$$

$$\int^M \mathfrak{A}^1 = \int^M \mathfrak{A}^0 \Rightarrow \mathfrak{A}^1 - \mathfrak{A}^0 = d\mathfrak{A}$$

$$\iota_{X^t} \mathfrak{A}^t = -\mathfrak{A}$$

$$\begin{aligned} -\left({}_2\mathfrak{b}_h \cdots {}_n\mathfrak{b}_h\right)^h \mathfrak{A} &= \left(X_h^t : {}_2\mathfrak{b}_h \cdots {}_n\mathfrak{b}_h\right)^h \mathfrak{A}^t = \left(X_h^t : {}_2\mathfrak{b}_h \cdots {}_n\mathfrak{b}_h\right) \left(\underbrace{1-t}_{\text{h}} \mathfrak{A}^1 + t \mathfrak{A}^0\right) \\ &= \underbrace{1-t}_{\text{h}} \left(X_h^t : {}_2\mathfrak{b}_h \cdots {}_n\mathfrak{b}_h\right)^h \mathfrak{A}^1 + t \left(X_h^t : {}_2\mathfrak{b}_h \cdots {}_n\mathfrak{b}_h\right)^h \mathfrak{A}^0 \\ &= \left(\underbrace{1-t}_{\text{h}} X_h^t : {}_2\mathfrak{b}_h \cdots {}_n\mathfrak{b}_h\right)^h \mathfrak{A}^1 + \left(t X_h^t : {}_2\mathfrak{b}_h \cdots {}_n\mathfrak{b}_h\right)^h \mathfrak{A}^0 \end{aligned}$$

$$\begin{aligned} \frac{d}{dt} \underbrace{{}_0\mathfrak{h}^t \times \mathfrak{A}^t}_{\text{h}} &= {}_0\mathfrak{h}^t \times \left(\frac{d}{dt} \mathfrak{A}^t + X^t \times \mathfrak{A}^t\right) = {}_0\mathfrak{h}^t \times \left(\mathfrak{A}^1 - \mathfrak{A}^0 + \underbrace{\iota_{X^t} \frac{d\mathfrak{A}^t}{dt}}_{=0} + d\overline{\iota_{X^t} \mathfrak{A}^t}\right) \\ &= {}_0\mathfrak{h}^t \times \left(\mathfrak{A}^1 - \mathfrak{A}^0 + d\overline{\iota_{X^t} \mathfrak{A}^t}\right) = {}_0\mathfrak{h}^t \times (d\mathfrak{A} - d\mathfrak{A}) = 0 \end{aligned}$$