$$T_{\widetilde{g}\left(u_\ell\right)} S_\ell \xrightarrow[k]{T_{u_\ell}(\widetilde{g})} T_{u_\ell} S_\ell$$

$$k\, u_\ell = \widetilde{g}\,\left(u_\ell\right)$$

$$J_u\left(\widetilde{g}\right) = \overbrace{\det k^{-1} T_{u_\ell}\left(\widetilde{g}\right)} \text{ well-def}$$

$$k\, u_\ell = k_1\, u_\ell \Longrightarrow k_1 = kh: \quad h\, u_\ell = u_\ell$$

$$k_1^{-1}\, T_{u_\ell}\left(\widetilde{g}\right) = h^{-1}\, k^{-1}\, T_{u_\ell}\left(\widetilde{g}\right)$$

$$\partial_t^0\, J_u\left(\widetilde{g}_t\right) = \text{ Re tr } T_{u_\ell}\left(\widetilde{\gamma}_\zeta\right)$$

$$\partial_t^0 \det k_t^{-1}\, T_{u_\ell}\left(\widetilde{g}_t\right) = \text{ tr } \partial_t^0\, k_t^{-1}\, T_{u_\ell}\left(\widetilde{g}_t\right) = \text{ tr } \underbrace{\partial_t^0 T_{u_\ell}\left(\widetilde{g}_t\right) - \dot{k}}$$

$$= \text{ tr } \underbrace{T_{u_\ell}\left(\partial_t^0 \widetilde{g}_t\right) - \dot{k}} = \text{ tr } \underbrace{T_{u_\ell}\left(\widetilde{\gamma}_\zeta\right) - \dot{k}}$$

$$\partial_t^0\, J_u\left(\widetilde{g}_t\right) = \text{ Re } \partial_t^0 \det k_t^{-1}\, T_{u_\ell}\left(\widetilde{g}_t\right) = \text{ Re tr } \underbrace{T_{u_\ell}\left(\widetilde{\gamma}_\zeta\right) - \dot{k}} = \text{ Re tr } T_{u_\ell}\left(\widetilde{\gamma}_\zeta\right)$$