

$${}^{x:y}\gamma = e^y (x^2 + 2xy)$$

$${}_x\underline{\gamma} = 2e^y (x + y) = 0 \Leftrightarrow y = -x$$

$${}_y\underline{\gamma} = e^y (x^2 + 2xy + 2x) = 0 \Leftrightarrow 0 = x^2 + 2xy + 2x = x^2 - 2x^2 + 2x = 2x - x^2 = x(x - 2)$$

$$\text{crit } (0:0) / (2: - 2)$$

$${}_{xx}\underline{\gamma} = 2e^y / {}_{yy}\underline{\gamma} = e^y (x^2 + 2xy + 4x) / {}_{xy}\underline{\gamma} = 2e^y (x + y + 1)$$

$$\Rightarrow H_\gamma (0:0) = \frac{2}{2} \Big| \frac{2}{0} / \det < 0 \text{ saddle} / H_\gamma (2: - 2) = \frac{2}{2} \Big| \frac{2}{4} > 0 \text{ min}$$

Flachendiskussion/crit pts/type/loc extr/max/min/saddle/glob extr?

$$5x^2 + y^2 + 10x - 6y + 6: \quad xy^2 + x^2 + 2y^2: \quad (4x^2 - y^2) e^{-x^2}: \quad \sin x \sin y: \quad x^4 - 2x^2y + x^2 + y^2$$

$$x^2 + 1 - 2x(\sin y + \cos y): \quad x^3 + y^3: \quad x + y + \sqrt{1 - x^2 - y^2}: \quad (x^2 + y^2 - 1)^2 \text{ Higgs-Potential/sketch graph}$$

NiveauFlachen-Partition

$$x^2 + y^2 + z^2 \text{ on } \mathbb{R}^3$$

krit Pts/Extrema/Niveaulinien-Partition

$$1 - x^2 - 2y^2: \quad x^3 - 3xy^2: \quad x^2 - y^2$$

$$3xe^y - x^3 - e^{3y} \text{ surj}$$

$$x^2 + \alpha xy + y^2 \text{ in } 0:0 \begin{cases} \text{loc isol min} & \overline{\alpha} < 2 \\ \text{nicht-isol loc min} & \overline{\alpha} = 2: \quad \alpha > 2 \Rightarrow \text{extrema on } B_1(0) \\ \text{kein loc extr} & \overline{\alpha} > 2 \end{cases}$$

loc extr/Sattelpt on \mathbb{R}^3

$$x^3 + 2y^2 - 2xy - 2x + 2y + 1$$

$${}^{x:y}\gamma = x^4 + x^2 - 2x^2y + y^2 \begin{cases} \text{loc extrema} \\ \text{Bild } \mathbb{R}^2 \gamma \end{cases}$$

$3xy$ max/min on $x^2 + y^2 \leq 2$

$$\begin{aligned} x\eta = 3y = 0: \quad y\eta = 3x = 0 &\Rightarrow x = 0 = y \\ y = \sqrt{2 - x^2} &\Rightarrow x\eta = 3x\sqrt{2 - x^2} \\ x\eta = 3\sqrt{2 - x^2} - \frac{3x^2}{\sqrt{2 - x^2}} = 0 &\Rightarrow 2 - x^2 = x^2 \Rightarrow x = \pm 1 \end{aligned}$$

$4x^2 + 3xy/x^2 + 2y^2 - x$ max/min on $x^2 + y^2 \leq 1$

$$x:y\eta = x^2 + 2y^2 - x$$

$$\text{Inner } 0:0 = \nabla\eta = (2x - 1:4y) \Rightarrow \begin{cases} x = 1/2 \\ y = 0 \end{cases} \quad 1/2:0\eta = -\frac{1}{4} \text{ min}$$

$$\text{Rand } y^2 = 1 - x^2 \Rightarrow x\tilde{\eta} = x^2 + 2(1 - x^2) - x = 2 - x^2 - x \Rightarrow 0 = x\tilde{\eta} = -2x - 1$$

$$\Rightarrow \begin{cases} x = -1/2 \\ y = \pm\sqrt{3}/2 \end{cases} \quad -1/2:\pm\sqrt{3}/2\eta = \frac{9}{4} \text{ max}$$

$x^3 + 3xy^2$ max/min on $x^2 + y^2 = 1$

$$P = \frac{x:y \in \mathbb{R}^2}{y = x^2}: K = \frac{u:v \in \mathbb{R}^2}{(u-3)^2 + v^2 = 1} \text{ disj/Abst } \inf_{x:y \in P: u:v \in K} \frac{\overline{|x:y - u:v|}}{\quad} \text{ (Abstand-Quadrat besser)}$$

Def-Ber/Hesse-Matrix

$$e^{x/y} \cos(xy) + e^x \sin y: \quad \log \frac{1+x}{1+y} - e^{xyz}: \quad \sqrt{x^2 + y^2 - z^2}$$

$$x:y\eta = \frac{2x^2 + 3y^2}{x^2 + y^2} \Rightarrow \bigwedge_{t > 0} tx:ty\eta = x:y\eta \Rightarrow \bigvee_{\text{max/min}}^{\mathbb{R}^2 \setminus 0} \text{ global/Bestimme}$$

$$\mathbb{R}^2 \xrightarrow[\text{diff}]{\gamma} \mathbb{R}: \quad \text{Nst } \frac{p \in \mathbb{R}^2}{p\gamma = 0} \ni o \text{ isoliert} \Rightarrow \underline{\gamma} = 0$$

Widerspruchsbeweis/SIF

$x^2 y^2$ loc Extr on $S_1(0)$

$$\mathbb{R}^n \supset G \text{ off/bes} : \begin{cases} \bar{G}/G \xrightarrow[\text{stet/diff}]{\gamma} \mathbb{R} \\ \gamma|_{\partial G} = 0 \end{cases} \Rightarrow \bigvee_{a \in G} \nabla^a \gamma = 0$$

$$p(x:y) = 4x^3 + 3xy - y^2 + 3 \quad \text{Nst on } \mathbb{S}^1 \cup \frac{x:0}{|x| \leq 1}$$

$$\text{extr}_{\mathbb{S}^n} x_0^2 \cdots x_n^2$$