$$n = \sum_{j}^{1|n} j\,k_j = k_1 + 2\,k_2 + \cdot\cdot + n\,k_n \text{ conjugacy type}$$

$$\mathsf{C}_k(n) = \frac{\pi \in \mathsf{C}(n)}{\pi \text{ hat } k_j \text{ disj j-cycles}}$$

$$\pi \in \mathsf{C}_k(n) \Leftrightarrow \pi = \prod_i^{k_1} \gamma_i^1 \cdots \prod_i^{k_n} \gamma_i^n = \prod_j^n \prod_i^{k_j} \gamma_i^j \text{ disj j-cycles}$$

$$k_j \text{ cycles } \gamma_i^j \text{ length } j$$

$$\sharp\,\mathsf{C}_k(n) = \frac{n!}{k_1!1^{k_1}k_2!2^{k_2}\cdots k_n!n^{k_n}} = \prod_j^{1|n} \frac{j}{k_j!j^{k_j}}$$

$$\pi \in \mathsf{C}(n) \xrightarrow[\text{surj}]{} \mathsf{C}_k(n) \ni {}_k\pi$$

$$\pi = \pi_1\,\pi_2 \cdots \pi_n$$

$${}_k\pi = \bar\pi_1\,\bar\pi_2 \cdot\cdot \bar\pi_{k_1} \,|\overline{\pi_{k_1+1|2}\cdots\pi_{k_1+2k_2-1|0}}|\overline{\pi_{k_1+2k_2+1|3}\cdots\pi_{k_1+2k_2+3k_3-2|0}}|\cdot\cdot|$$

$$\overline{\pi_{k_1+\cdot\cdot+(n-1)\,k_{n-1}+1|n}\cdots\pi_{k_1+\cdot\cdot+nk_n-(n-1)\,|0}}$$

$$= \bar\pi_1\,\bar\pi_2 \cdot\cdot \bar\pi_{k_1} |\overline{\pi_{k_1}^{+1|2}\cdots\pi_{k_1+2k_2}^{-1|0}}|\overline{\pi_{k_1+2k_2}^{+1|3}\cdots\pi_{k_1+2k_2+3k_3}^{-2|0}}|\cdot\cdot|\overline{\pi_{k_1+\cdot\cdot+(n-1)\,k_{n-1}}^{+1|n}\cdots\pi_{k_1+\cdot\cdot+nk_n}^{-(n-1)\,|0}}$$

$$\text{k-equivalence } {}_k\sigma = {}_k\tau \Leftrightarrow \begin{cases} \text{external permutation i-cycles} & k_1!k_2!\cdots k_n! \\ \text{internal cyclic permutation i-cycles} & 1^{k_1}2^{k_2}\cdot\cdot n^{k_n} \end{cases}$$

$$\Leftrightarrow \begin{cases} \text{j cyclic perms in each j-cycle} & \Rightarrow j^{k_j} \text{ choices} \\ k_j! \text{ perms of all j-cycles} & \Rightarrow k_j! \text{ choices} \end{cases}$$

$$\Rightarrow \frac{n!}{\sharp\mathsf{C}_k(n)} = \sharp\frac{\mathsf{C}(n)}{\mathsf{C}_k(n)} = k_1!\,1^{k_1}\,k_2!\,2^{k_2}\cdots k_n!\,n^{k_n} \Rightarrow \text{ Beh}$$