

$$\xrightarrow{\partial} \underbrace{\mathbb{K}\langle X \rangle}_{n+1} \xrightarrow{\partial} \underbrace{\mathbb{K}\langle X \rangle}_{n} \xrightarrow{\partial} \underbrace{\mathbb{K}\langle X \rangle}_{1} \xrightarrow{\partial} \underbrace{\mathbb{K}\langle X \rangle}_{0} \xrightarrow{\text{on}} \mathbb{K}$$

proj resolution of \mathbb{K}

$$\begin{aligned} \underbrace{\mathbb{K}\langle X \rangle}_{n} &= \sum_{0 \leq i \leq n} -1 \underbrace{\mathbb{K}\langle X \rangle}_{i} \otimes \mathbb{K}\langle X \rangle_{0} \otimes \dots \otimes \mathbb{K}\langle X \rangle_{i} \otimes \dots \otimes \mathbb{K}\langle X \rangle_{n} \\ + \sum_{0 \leq i < j \leq n} & -1 \underbrace{\mathbb{K}\langle X \rangle}_{i} \otimes \dots \otimes \mathbb{K}\langle X \rangle_{j} \otimes \dots \otimes \mathbb{K}\langle X \rangle_{i} \otimes \dots \otimes \mathbb{K}\langle X \rangle_{j} \otimes \dots \otimes \mathbb{K}\langle X \rangle_{n} \end{aligned}$$

$$\underbrace{\mathbb{K}\langle X \rangle}_{j} = \mathbb{K}\langle X \rangle \otimes \mathbb{K}\langle X \rangle$$

$$\underbrace{\mathbb{K}\langle X \rangle}_{j} = \text{tr } \mathbb{K}\langle X \rangle$$