$$
\begin{array}{c|c|c|c}
q^1 & 0 & 0 & 0 \\ \hline
0 & q^i & 0 & 0 \\ \hline
0 & 0 & q^j & 0 \\ \hline
0 & 0 & 0 & q^n
\end{array}
\times
\begin{array}{c|c|c|c}
{}_1p & \dfrac{1}{q^1-q^i} & \dfrac{1}{q^1-q^j} & \dfrac{1}{q^1-q^n} \\ \hline
\dfrac{1}{q^i-q^1} & {}_ip & \dfrac{1}{q^i-q^j} & \dfrac{1}{q^i-q^n} \\ \hline
\dfrac{1}{q^j-q^1} & \dfrac{1}{q^j-q^i} & {}_jp & \dfrac{1}{q^j-q^n} \\ \hline
\dfrac{1}{q^n-q^1} & \dfrac{1}{q^n-q^i} & \dfrac{1}{q^n-q^j} & {}_np
\end{array}
= I - 1\,1^*
$$

$$
{}^i\mathrm{LHS}_i = q^i\,{}_ip - {}_ip\,q^i = 0 = {}^i\big(I - 11^*\big)_i
$$

$$
{}^i\mathrm{LHS}_j = \frac{q^i}{q^j - q^i} - \frac{q^j}{q^j - q^i} = -1 = {}^i\big(I - 11^*\big)_j
$$

$$
Q \times P = I - 1\,1^* \implies \bigvee_g^{{}^n\mathbb{C}_n^{\mathrm{U}}} gQ\,g^{-1} =
\begin{array}{c|c|c|c}
q^1 & 0 & 0 & 0 \\ \hline
0 & q^i & 0 & 0 \\ \hline
0 & 0 & q^j & 0 \\ \hline
0 & 0 & 0 & q^n
\end{array}
$$

$$
g1 = \begin{array}{c}
v^1 \\ \hline
v^i \\ \hline
v^j \\ \hline
v^n
\end{array}
$$

$$
\begin{array}{c|c|c|c}
q^1 & 0 & 0 & 0 \\ \hline
0 & q^i & 0 & 0 \\ \hline
0 & 0 & q^j & 0 \\ \hline
0 & 0 & 0 & q^n
\end{array}
\times \underbrace{gPg^{-1}} = I - (g1)\,(g1)^*
$$

$$
\mathrm{LHS} = \underbrace{gQg^{-1}} \times \underbrace{gPg^{-1}} = g\underbrace{Q \times P}\,g^{-1} = g\left(I - 11^*\right)\,g^{-1} = \mathrm{RHS}
$$

$$v^i\,\bar{v}^i = 1$$

$$1 - v^i\,\bar{v}^i = {}^i\!\left(I - (g1)\,(g1)^*\right)_i = {}^i\!\left(\begin{array}{c|c|c|c} q^1 & 0 & 0 & 0 \\ \hline 0 & q^i & 0 & 0 \\ \hline 0 & 0 & q^j & 0 \\ \hline 0 & 0 & 0 & q^n \end{array} \times \underbrace{gPg^{-1}} \right)_i = q^i\,{}^i\!\left(gPg^{-1}\right)_i - {}^i\!\left(gPg^{-1}\right)_i\,q^i = 0$$

$$\gamma = \begin{array}{c|c|c|c} \bar{v}^1 & 0 & 0 & 0 \\ \hline 0 & \bar{v}^i & 0 & 0 \\ \hline 0 & 0 & \bar{v}^j & 0 \\ \hline 0 & 0 & 0 & \bar{v}^n \end{array} \in {}_n\mathbb{T} \subset {}^n\mathbb{C}_n^{\cup}$$

$$\gamma\,(g1) = \begin{array}{c|c|c|c} \bar{v}^1 & 0 & 0 & 0 \\ \hline 0 & \bar{v}^i & 0 & 0 \\ \hline 0 & 0 & \bar{v}^j & 0 \\ \hline 0 & 0 & 0 & \bar{v}^n \end{array} \begin{array}{c} v^1 \\ \hline v^i \\ \hline v^j \\ \hline v^n \end{array} = \begin{array}{c} 1 \\ \hline 1 \\ \hline 1 \\ \hline 1 \end{array} = 1$$

$$\pi \in {}_n\mathcal{S} \subset {}^n\mathbb{C}_n^{\cup}: \quad q^{\pi_1} < \cdot\cdot < q^{\pi_n}$$

$$\pi 1 = 1$$

$$\pi\gamma g \in G_{I-11^*}$$

$$\pi\gamma g 1 = \pi\gamma\,(g1) = \pi 1 = 1$$

$$\underbrace{\pi\gamma g}\,\left(I - 11^*\right)\,\overbrace{\pi\gamma g}^{-1} = I - \underbrace{\pi\gamma g 1}\,\overbrace{\pi\gamma g 1}^{*} = I - 1\,1^*$$

$$\underbrace{\pi\gamma g}\,Q\,\overbrace{\pi\gamma g}^{-1} = \begin{array}{c|c|c|c} q^{\pi_1} & 0 & 0 & 0 \\ \hline 0 & q^{\pi_i} & 0 & 0 \\ \hline 0 & 0 & q^{\pi_j} & 0 \\ \hline 0 & 0 & 0 & q^{\pi_n} \end{array}$$

$$\text{LHS} = \pi\gamma\,\underbrace{gQg^{-1}}\,\gamma^{-1}\,\pi^{-1} = \pi\gamma \begin{array}{c|c|c|c} q^1 & 0 & 0 & 0 \\ \hline 0 & q^i & 0 & 0 \\ \hline 0 & 0 & q^j & 0 \\ \hline 0 & 0 & 0 & q^n \end{array} \gamma^{-1}\,\pi^{-1} = \pi \begin{array}{c|c|c|c} q^1 & 0 & 0 & 0 \\ \hline 0 & q^i & 0 & 0 \\ \hline 0 & 0 & q^j & 0 \\ \hline 0 & 0 & 0 & q^n \end{array} \pi^{-1} = \text{RHS}$$

$$
gPg^{-1} = \begin{array}{c|c|c|c}
{}_1p & \dfrac{1}{q^1-q^i} & \dfrac{1}{q^1-q^j} & \dfrac{1}{q^1-q^n} \\
\hline
\dfrac{1}{q^i-q^1} & {}_ip & \dfrac{1}{q^i-q^j} & \dfrac{1}{q^i-q^n} \\
\hline
\dfrac{1}{q^j-q^1} & \dfrac{1}{q^j-q^i} & {}_jp & \dfrac{1}{q^j-q^n} \\
\hline
\dfrac{1}{q^n-q^1} & \dfrac{1}{q^n-q^i} & \dfrac{1}{q^n-q^j} & {}_np
\end{array}
$$

$$
0 = {}^{i}\big(I - 11^*\big)_{i} = {}^{i}\left(\begin{array}{c|c|c|c} q^1 & 0 & 0 & 0 \\ \hline 0 & q^i & 0 & 0 \\ \hline 0 & 0 & q^j & 0 \\ \hline 0 & 0 & 0 & q^n \end{array} \times \underbrace{gPg^{-1}} \right)_{i} = q^i\, {}^{i}\big(gPg^{-1}\big)_{i} - {}^{i}\big(gPg^{-1}\big)_{i}\, q^i \Longrightarrow {}^{i}\big(gPg^{-1}\big)_{i} = {}_{i}p \text{ bel}
$$

$$
-1 = {}^{i}\big(I - 11^*\big)_{j} = {}^{i}\left(\begin{array}{c|c|c|c} q^1 & 0 & 0 & 0 \\ \hline 0 & q^i & 0 & 0 \\ \hline 0 & 0 & q^j & 0 \\ \hline 0 & 0 & 0 & q^n \end{array} \times \underbrace{gPg^{-1}} \right)_{j} = q^i\, {}^{i}\big(gPg^{-1}\big)_{j} - {}^{i}\big(gPg^{-1}\big)_{j}\, q^j \Longrightarrow {}_{i}p = {}^{i}\big(gPg^{-1}\big)_{j} \underbrace{q^i - q^j}
$$

$$
\Longrightarrow {}^{i}\big(gPg^{-1}\big)_{j} = \frac{1}{q^j - q^i}
$$