$$_{q:p}H_2 = \sum_i \frac{{}_i p^2}{2} + \sum_{i<j} \frac{1}{\left(q^i - q^j\right)^2}$$

$$H_2 = \frac{1}{2} \text{ tr } P^2$$

$$2 \text{ RHS } = \text{ tr } \left(\begin{array}{c|c|c|c} {}_1p & \frac{1}{q^1 - q^i} & \frac{1}{q^1 - q^j} & \frac{1}{q^1 - q^n} \\ \hline \frac{1}{q^i - q^1} & {}_ip & \frac{1}{q^i - q^j} & \frac{1}{q^i - q^n} \\ \hline \frac{1}{q^j - q^1} & \frac{1}{q^j - q^i} & {}_jp & \frac{1}{q^j - q^n} \\ \hline \frac{1}{q^n - q^1} & \frac{1}{q^n - q^i} & \frac{1}{q^n - q^j} & {}_np \end{array} \right)^2 = \sum_i {}_ip^2 + \sum_{i \neq j} \frac{1}{\left(q^i - q^j\right)^2} = 2 \text{ LHS}$$