$$\partial_v \times \partial_{\lambda x} = \partial_{\lambda v}$$

$$\partial_v \partial_{\lambda x} {}^x\mathcal{E}_b^{m+} = \partial_v \overbrace{\underbrace{\lambda x} \bowtie b\, {}^x\mathcal{E}_b^{m}} = \partial_v \overbrace{x \bowtie \underbrace{\overset{*}{\lambda} b}\, {}^x\mathcal{E}_b^m} = \overbrace{v \bowtie \underbrace{\overset{*}{\lambda} b}}\, {}^x\mathcal{E}_b^m + \overbrace{x \bowtie \underbrace{\overset{*}{\lambda} b}}\underbrace{v \bowtie b}\, {}^x\mathcal{E}_b^{m-}$$

$$= \overbrace{\underbrace{\lambda v} \bowtie b}\, {}^x\mathcal{E}_b^m + \overbrace{\underbrace{\lambda x} \bowtie b}\underbrace{v \bowtie b}\, {}^x\mathcal{E}_b^{m-}$$

$$\partial_{\lambda x}\partial_v {}^x\mathcal{E}_b^{m+} = \partial_{\lambda x} \underbrace{v\bowtie b}\,{}^x\mathcal{E}_b^m = \underbrace{v\bowtie b}\,\partial_{\lambda x}{}^x\mathcal{E}_b^m = \underbrace{v\bowtie b}\underbrace{\lambda x\bowtie b}\,{}^x\mathcal{E}_b^{m-}$$

$$\overbrace{\partial_v\times\partial_{\lambda x}}\,{}^x\mathcal{E}_b^{m+} = \overbrace{v \bowtie \underbrace{\overset{*}{\lambda} b}}\,{}^x\mathcal{E}_b^m = \partial_{\lambda v}{}^x\mathcal{E}_b^{m+}$$

$$A \times \underbrace{BC} = \underbrace{A\times B}C + B\underbrace{A\times C}$$

$$\text{RHS} = \underbrace{AB - BA}C + B\underbrace{AC - CA} = ABC - BCA = \text{LHS}$$

$$\partial_v \times \underbrace{\partial_\lambda\partial_w} = \partial_{\lambda v}\partial_w$$

$$\text{LHS} = \overbrace{\partial_v\times\partial_\lambda}\,\partial_w + \partial_\lambda\overbrace{\partial_v\times\partial_w}^{=0} = \text{RHS}$$

$$\underbrace{AB}\times\underbrace{CD} = \underbrace{A\times C}BD + A\underbrace{B\times C}D + C\underbrace{A\times D}B + CA\underbrace{B\times D}$$

$$\text{LHS} = \overbrace{\underbrace{AB}\times C}D + C\overbrace{\underbrace{AB}\times D} = -\overbrace{C\times\underbrace{AB}}D - C\overbrace{D\times\underbrace{AB}} = -\overbrace{\underbrace{C\times A}B + A\underbrace{C\times B}}D$$

$$-C\overbrace{\underbrace{D\times A}B + A\underbrace{D\times B}} = \overbrace{\underbrace{A\times C}B + A\underbrace{B\times C}}D + C\overbrace{\underbrace{A\times D}B + A\underbrace{B\times D}} = \text{RHS}$$

$$\overbrace{\partial_\lambda \partial_v} \times \overbrace{\partial_\mu \partial_w} = \partial_{\lambda \times \mu}\,\partial_v\,\partial_w + \partial_\lambda\,\partial_{\mu v}\,\partial_w - \partial_\mu\,\partial_{\lambda w}\,\partial_v$$

$$\text{LHS} = \underbrace{\partial_\lambda \times \partial_\mu}\,\partial_v\,\partial_w + \partial_\lambda\,\underbrace{\partial_v \times \partial_\mu}\,\partial_w + \partial_\mu\,\underbrace{\partial_\lambda \times \partial_w}\,\partial_v + \partial_\mu\,\partial_\lambda\,\underbrace{\partial_v \times \partial_w}_{=0} = \text{RHS}$$

$$\mathcal{B}_w = \frac{\ell a}{2}\,\partial_w + \partial_{w \overset{*}{e}_i x}\,\partial_{e_i}$$

$$\mathcal{B}_v \times \mathcal{B}_w = \left(\frac{\ell a}{2}\partial_v + \partial_{v \overset{*}{e}_i x}\partial_{e_i}\right) \times \left(\frac{\ell a}{2}\partial_w + \partial_{w \overset{*}{e}_j x}\partial_{e_j}\right)$$

$$= \left(\frac{\ell a}{2}\right)^2 \overbrace{\partial_v \times \partial_w}^{=0} + \frac{\ell a}{2}\left(\partial_v \times \overbrace{\partial_{w \overset{*}{e}_j x}\partial_{e_j}} + \overbrace{\partial_{v \overset{*}{e}_i x}\partial_{e_i}} \times \partial_w\right) + \overbrace{\partial_{v \overset{*}{e}_i x}\partial_{e_i}} \times \overbrace{\partial_{w \overset{*}{e}_j x}\partial_{e_j}}$$

$$= \frac{\ell a}{2}\left(\partial_v \times \overbrace{\partial_{w \overset{*}{e}_j x}\partial_{e_j}} - \partial_w \times \overbrace{\partial_{v \overset{*}{e}_i x}\partial_{e_i}}\right) + \overbrace{\partial_{v \overset{*}{e}_i x}\partial_{e_i}} \times \overbrace{\partial_{w \overset{*}{e}_j x}\partial_{e_j}}$$

$$= \frac{\ell a}{2}\overbrace{\partial_{w \overset{*}{e}_j v}\partial_{e_j} - \partial_{v \overset{*}{e}_i w}\partial_{e_i}}^{=0} + \partial_{v \overset{*}{e}_i \times w \overset{*}{e}_j}\,\partial_{e_i}\,\partial_{e_j} + \partial_{v \overset{*}{e}_i x}\,\partial_{w \overset{*}{e}_j e_i}\,\partial_{e_j} - \partial_{w \overset{*}{e}_j x}\,\partial_{v \overset{*}{e}_i e_j}\,\partial_{e_i}$$