

$$\begin{aligned}
\overbrace{[1 \cdots m+n]}^{\prime} \mathbb{I} &= \sum_{\begin{cases} \sigma_1 < \cdots < \sigma_m \\ \sigma_{m+1} < \cdots < \sigma_{m+n} \end{cases}} (-1)^\sigma \overbrace{[\sigma_1 \cdots \sigma_m]}^{\prime} \overbrace{[\sigma_{m+1} \cdots \sigma_{m+n}]}^{\prime} \\
&= \frac{1}{m!n!} \sum_{\pi} (-1)^\pi \overbrace{[\pi_1 \cdots \pi_m]}^{\prime} \overbrace{[\pi_{m+1} \cdots \pi_{m+n}]}^{\prime}
\end{aligned}$$