$$\mathsf{C}|n = n! \text{ perms of } n$$

$$\pi \in \mathsf{C}|n \text{ Fixpunkt } i \in n \Leftrightarrow \pi(i) = i$$

$$1|n = \text{ Menge der Eigenschaften}$$

$$\text{property } i = \text{ hat Fixpoint } i$$

$$A_i = \begin{cases} \pi \in \mathsf{C}|n \\ \pi(i) = i \end{cases}$$

$$F(\pi) = \text{ fix points}$$

$$A_S^{\supset} = \frac{\pi \in \mathsf{C}|n}{F(\pi) \supset S} = \frac{\pi \in \mathsf{C}|n}{\pi|S = \imath}$$

$$|A_S^{\supset}| = (n - |S|)!$$

$$\pi|S = \imath \Longrightarrow n \llcorner S \xrightarrow[\text{bel perm}]{\pi} n \llcorner S \Longrightarrow (n - |S|)! \text{ choices}$$

$$A_S^{=} = \frac{\pi \in \mathsf{C}|n}{F(\pi) = S} = \frac{\pi \in \mathsf{C}|n}{\pi|S = \imath: \quad \pi|n \llcorner S \text{ no fix point}}$$

$$A_\varnothing^{=} = \frac{\pi \in \mathsf{C}|n}{F(\pi) = \varnothing} = \frac{\pi \in \mathsf{C}|n}{\pi|n \text{ no fix point}}$$

$$\frac{\sharp \text{ fixpoint-free perms=derangements}}{n!} = \sum_i^{0|n} \frac{(-1)^i}{i!}$$

$$\text{Moe} \quad \underbrace{|A_\varnothing^{=}|}_{\text{no fixpoint}} = \sum_{T \subset N} (-1)^T |A_T^{\supset}| = \sum_{T \subset N} (-1)^T (n - |T|)!$$

$$= \sum_i^{0|n} \sum_{T \subset N}^{|T| = i} (-1)^i (n-i)! = \sum_i^{0|n} \begin{bmatrix} n \\ i \end{bmatrix} (-1)^i (n-i)! = \sum_i^{0|n} \frac{n!}{i!} (-1)^i$$