

$$\begin{aligned}
& M^4 \\
& \dim \Lambda^2(\underline{M}) = \begin{bmatrix} 4 \\ 2 \end{bmatrix} = 6 \\
& \Lambda^2(\underline{M}) \xrightarrow[\text{Hodge}]{*} \Lambda^2(\underline{M}) \\
& *^2 = -1 \\
& \Lambda^2(\underline{M}) = \Lambda_+^2(\underline{M}) \times \Lambda_-^2(\underline{M}) \\
& \Lambda_\pm^2(\underline{M}) \ni e_1 \mathbf{x} e_2 \pm e_3 \mathbf{x} e_4 \\
& \dim \Lambda_\pm^2(\underline{M}) = 3 \\
& R = R_{ijkl} \\
& \Lambda^2(\underline{M}) \xleftarrow{R} \Lambda^2(\underline{M}) \\
& R \widehat{e_i \mathbf{x} e_j} = R_{ijkl} e_k \mathbf{x} e_l \\
& R = \frac{A_+}{B^*} \Big| \frac{B}{A_-} = \frac{W_+}{B^*} \Big| \frac{B}{W_-} + \frac{\tau}{12} \\
& W_\pm = A_\pm + \frac{\tau}{12}
\end{aligned}$$

$$\text{tr } W_\pm = 0$$

$$\begin{aligned}
& \text{tr } A_\pm = -\frac{\tau}{4} \\
& \text{tr } W_\pm = \text{tr } A_\pm + 3\frac{\tau}{12} = -\frac{\tau}{4} + 3\frac{\tau}{12} = 0
\end{aligned}$$

$$M:g \text{ Einstein} \Leftrightarrow B = 0$$

$$M:g \text{ self-dual} \Leftrightarrow W_- = 0$$

$$M:g \text{ anti-dual} \Leftrightarrow W_+ = 0$$