$$\mathbb{C}\overset{2}{\triangle}_{\omega}\mathbb{C} \overset{\ltimes}{\longleftarrow} \operatorname{Aff}\mathbb{C} \ltimes \mathbb{C}\overset{2}{\triangle}_{\omega}\mathbb{C}$$

$$z|1\begin{array}{c|c} a & 0 \\ \hline b & 1 \end{array} = za+b|1$$

$$\overbrace{{}^{z}\begin{array}{c|c} a & 0 \\ \hline b & 1 \end{array}\urcorner} = {}^{za+b}\urcorner$$

$$g \ltimes \underbrace{\acute{g}\ltimes\urcorner} = \underbrace{g\acute{g}}\ltimes\urcorner$$

$$\overbrace{{}^{z}\begin{array}{c|c} 1 & 0 \\ \hline b & 1 \end{array}\urcorner} = {}^{z+b}\urcorner$$

$$\begin{array}{c|c} 1 & 0 \\ \hline b & 1 \end{array}\ \mathcal{E}_w = \mathfrak{e}^{b\bar{w}}\,\mathcal{E}_w$$

$$\overbrace{{}^{z}\begin{array}{c|c} 1 & 0 \\ \hline b & 1 \end{array}\ \mathcal{E}_w} = {}^{z+b}\mathcal{E}_w = \mathfrak{e}^{(z+b)\,\bar{w}} = \mathfrak{e}^{b\bar{w}}\,\mathfrak{e}^{z\bar{w}} = \mathfrak{e}^{b\bar{w}}\,{}^{z}\mathcal{E}_w$$

$$\overbrace{{}^{z}\begin{array}{c|c} 1 & \overset{*}{0} \\ \hline b & 1 \end{array}\urcorner} = \mathfrak{e}^{z\bar{b}}\,{}^{z}\urcorner$$

$$\overbrace{{}^{z}\begin{array}{c|c} 1 & \overset{*}{0} \\ \hline b & 1 \end{array}\urcorner} = \mathcal{E}_z \bowtie \underbrace{\begin{array}{c|c} 1 & \overset{*}{0} \\ \hline b & 1 \end{array}\urcorner} = \underbrace{\begin{array}{c|c} 1 & 0 \\ \hline b & 1 \end{array}\ \mathcal{E}_z}\bowtie\urcorner = \underbrace{\mathfrak{e}^{b\bar{z}}\mathcal{E}_z}\bowtie\urcorner = \mathfrak{e}^{z\bar{b}}\ \underbrace{\mathcal{E}_z\bowtie\urcorner} = \mathfrak{e}^{z\bar{b}}\ {}^{z}\urcorner$$

$${}^{z}\overline{b\ltimes\urcorner} = {}^{z+b}\urcorner\,\mathfrak{e}^{-b\bar{b}/2-z\bar{b}}$$

$$\overline{b\ltimes\urcorner}\bowtie\overline{b\ltimes\top} = \urcorner\bowtie\top$$

$$\text{LHS} = \int_{dz/\pi}^{\mathbb{C}} \mathfrak{e}^{-z\bar{z}}\ {}^{z}\overline{b\ltimes\urcorner}\ {}^{z}\overline{b\ltimes\top} = \int_{dz/\pi}^{\mathbb{C}} \mathfrak{e}^{-z\bar{z}}\ \overline{{}^{z+b}\urcorner\mathfrak{e}^{-b\bar{b}/2-z\bar{b}}}\ {}^{z+b}\top\,\mathfrak{e}^{-b\bar{b}/2-z\bar{b}}$$

$$= \mathfrak{e}^{-b\bar{b}}\int_{dz/\pi}^{\mathbb{C}} \mathfrak{e}^{-z\bar{z}}\ \mathfrak{e}^{-\bar{z}b-z\bar{b}}\ {}^{z+b}\bar{\urcorner}\ {}^{z+b}\top = \int_{dz/\pi}^{\mathbb{C}} \mathfrak{e}^{-(z+b)\overline{z+b}}\ {}^{z+b}\bar{\urcorner}\ {}^{z+b}\top = \int_{dw/\pi}^{\mathbb{C}} \mathfrak{e}^{-w\bar{w}}\ {}^{w}\bar{\urcorner}\ {}^{w}\top = \text{RHS}$$

$$a \ltimes \underbrace{b \ltimes \text{ʅ}} = \underbrace{a+b} \ltimes \text{ʅ}\,\mathfrak{e}^{\underbrace{b\bar{a} - a\bar{b}}/2}$$

$$ {}^{z}\overbrace{a \ltimes \underbrace{b \ltimes \text{ʅ}}} = {}^{z+a}\overbrace{b \ltimes \text{ʅ}}\;\mathfrak{e}^{-a\bar{a}/2 - z\bar{a}} = {}^{\overbrace{z+a}+b}\text{ʅ}\;\mathfrak{e}^{-b\bar{b}/2 - \overbrace{z+a}\,\bar{b}}\;\mathfrak{e}^{-a\bar{a}/2 - z\bar{a}}$$

$$= {}^{z+\overbrace{a+b}}\text{ʅ}\;\mathfrak{e}^{-\overbrace{a+b}\,\overline{a+b}/2 - z\overline{a+b}}\;\mathfrak{e}^{\underbrace{b\bar{a} - a\bar{b}}/2} = {}^{z}\overbrace{\underbrace{a+b} \ltimes \text{ʅ}}\;\mathfrak{e}^{\underbrace{b\bar{a} - a\bar{b}}/2}$$

$\mathbb{T}\times\mathbb{C} \ni \sigma{:}a$ central extension

$$\underbrace{\sigma|a}\,\underbrace{\tau|b} = \sigma\tau\,\mathfrak{e}^{\underbrace{b\bar{a} - a\bar{b}}/2}|\underbrace{a+b}$$

$$\underbrace{\sigma|a}\,\text{ʅ} = \sigma\,\underbrace{a \ltimes \text{ʅ}}$$

$$\underbrace{\sigma|a}\,\text{ʅ}\bar{\bowtie}\underbrace{\sigma|a}\,\text{ʇ} = \text{ʅ}\bar{\bowtie}\,\text{ʇ}$$

$$\underbrace{\sigma|a}\,\underbrace{\tau|b} = \sigma\tau\,\mathfrak{e}^{\underbrace{b\bar{a} - a\bar{b}}/2}|\underbrace{a+b} \text{ unit rep}$$

$$\underbrace{\sigma|a}\,\underbrace{\tau|b}\,\text{ʅ} = \underbrace{\sigma|a}\,\tau b \ltimes \text{ʅ} = \tau\,\underbrace{\sigma|a}\,\underbrace{b \ltimes \text{ʅ}} = \tau\sigma\,\overbrace{a \ltimes \underbrace{b \ltimes \text{ʅ}}} = \tau\sigma\,\overbrace{\underbrace{a+b} \ltimes \text{ʅ}}\,\mathfrak{e}^{\underbrace{b\bar{a} - a\bar{b}}/2} = \tau\sigma\,\mathfrak{e}^{\underbrace{b\bar{a} - a\bar{b}}/2}\,\overbrace{\underbrace{a+b} \ltimes \text{ʅ}}$$