

$$u = \frac{1 - z\bar{z}}{1 + z\bar{z}}$$

$$v = \frac{z + \bar{z}}{1 + z\bar{z}}$$

$$w = i \frac{\bar{z} - z}{1 + z\bar{z}}$$

$$u^2 + v^2 + w^2 = 1$$

$$\overbrace{1 - z\bar{z}}^2 + \overbrace{z + \bar{z}}^2 + \overbrace{i(z - \bar{z})}^2 = \overbrace{1 - z\bar{z}}^2 + \overbrace{z + \bar{z}}^2 - \overbrace{\bar{z} - z}^2 = \overbrace{1 - z\bar{z}}^2 + 4z\bar{z} = \overbrace{1 + z\bar{z}}^2$$

$$z\bar{z} = \frac{1 - u}{1 + u}$$

$$\frac{1 + z\bar{z}}{2} = 1 + u$$

$$z = \frac{v + iw}{1 + u}$$

$$\bar{z} = \frac{v - iw}{1 + u}$$