

$$\mathrm{M}/\mathbb{T}^k \text{ comp/no winding} : \quad \mathscr{X}=0$$

$$\frac{1}{2} \text{ BPS} \iff \text{rat}_q^p \left\{ \begin{array}{l} \frac{1}{2} \text{ BPS} \iff \text{ sphere } C \\ \text{dual } \frac{1}{2} \text{ BPS} \iff \text{ union } C \cup \tilde{C} = \text{ anti-canonical} \end{array} \right.$$

$$\text{brane tension} = \exp \overline{C}$$

$$\begin{bmatrix} \mathbb{H} \\ 4 \end{bmatrix} \boxminus \begin{bmatrix} k \\ 0 \end{bmatrix} = \begin{bmatrix} 1\mathbb{H}\text{-}k \\ 4 \end{bmatrix}$$

$$\mathbb{P}^2_k$$

$$\mathbb{Z}^{1:k} = \mathbb{Z} < C_0:C_1\cdots C_k >$$

$$\text{el} \, + \, \text{mg} \, = d-2 = 11-k-2 = 9-k$$

$$-K=3C_0-C_1-\cdots-C_k$$

$$\deg_C = -K|C$$

$$C+\tilde{C}=-K$$

$$p+q=\deg_C+\deg_{\tilde{C}}=-K|-K=K|K=9-k$$

$$\text{pos roots}$$

$$\begin{aligned}
3 \leq k \leq 9: \quad & \mathbb{4} \frac{-1}{8-k} = \mathbb{4} \frac{2}{5} \boxminus \frac{3}{k-3} = {}_{9-k}^0 \text{rat} \left\{ \begin{array}{l} C_0 - C_{ij\ell} \\ 2C_0 - C_{K \lrcorner ij\ell} \end{array} \right. \left\{ \begin{array}{l} \emptyset \\ \overline{* \emptyset}_{-ij\ell} \end{array} \right. \begin{array}{l} 0 \\ 9-k \end{array} \left\{ \begin{array}{l} M_2^+ / \mathbb{T}_{ij\ell} \\ M_5^+ / \mathbb{T}_{K \lrcorner ij\ell} \end{array} \right. \begin{array}{l} \text{wrap}_3 M2 \\ \text{wrap}_{k-3} M5 \end{array} \\
2 \leq k \leq 8: \quad & \mathbb{4} \frac{0}{7-k} = \mathbb{4} \frac{2}{5} \boxminus \frac{2}{k-2} = {}_{8-k}^1 \text{rat} \left\{ \begin{array}{l} C_0 - C_{ij} \\ 2C_0 - C_{K \lrcorner ij} \end{array} \right. \left\{ \begin{array}{l} \mathcal{X}_{ij} \\ \overline{* \mathcal{X}}_{-ij} \end{array} \right. \begin{array}{l} 1 \\ 8-k \end{array} \left\{ \begin{array}{l} M_2^+ / \mathbb{T}_{ij} \\ M_5^+ / \mathbb{T}_{K \lrcorner ij} \end{array} \right. \begin{array}{l} \text{wrap}_2 M2 \\ \text{wrap}_{k-2} M5 \end{array} \\
1 \leq k \leq 7: \quad & \mathbb{4} \frac{1}{6-k} = \mathbb{4} \frac{2}{5} \boxminus \frac{1}{k-1} = {}_{7-k}^2 \text{rat} \left\{ \begin{array}{l} C_0 - C_i \\ 2C_0 - C_{K \lrcorner i} \end{array} \right. \left\{ \begin{array}{l} \mathcal{Z}_i \\ \overline{* \mathcal{Z}}_{-i} \end{array} \right. \begin{array}{l} 2 \\ 7-k \end{array} \left\{ \begin{array}{l} M_2^+ / \mathbb{T}_i \\ M_5^+ / \mathbb{T}_{K \lrcorner i} \end{array} \right. \begin{array}{l} \text{wrap}_1 M2 \\ \text{wrap}_{k-1} M5 \end{array} \\
0 \leq k \leq 6: \quad & \mathbb{4} \frac{2}{5-k} = \mathbb{4} \frac{2}{5} \boxminus \frac{0}{k} = {}_{6-k}^3 \text{rat} \left\{ \begin{array}{l} C_0 \\ 2C_0 - C_K \end{array} \right. \left\{ \begin{array}{l} \mathcal{Z} \\ \overline{* \mathcal{Z}}_{-} \end{array} \right. \begin{array}{l} 3 \\ 6-k \end{array} \left\{ \begin{array}{l} M_2^+ \\ M_5^+ / \mathbb{T}_K \end{array} \right. \begin{array}{l} M2 \\ \text{wrap}_k M5 \end{array}
\end{aligned}$$

$$H \subset K: \quad H = (ij\ell): \quad H = (ij): \quad H = (i): \quad H = \emptyset$$

$$\overline{C_0 - C_H} | \overline{2C_0 - C_{K \lrcorner H}} = 2C_0 | C_0 = 2$$

$$-K | \overline{C_0 - C_H} = \overline{3C_0 - C_K} | \overline{C_0 - C_H} = \overline{3C_0 - C_H} | \overline{C_0 - C_H} = 3 - |H| = 0/1/2/3$$

KK related

$$\begin{aligned}
& \left\{ \begin{array}{l} 0 \\ 3C_0 - C_K = -K \end{array} \right. \begin{array}{l} 0 \\ 9-k \end{array} \left\{ \begin{array}{l} \emptyset \\ \overline{* \emptyset}_{-} \end{array} \right. \begin{array}{l} \text{dila}_e \\ \text{dila}_m \end{array} \\
& {}_{8-k}^1 \text{rat} \left\{ \begin{array}{l} C_i \\ 3C_0 - C_{K \lrcorner i} - 2C_i \end{array} \right. \begin{array}{l} 1 \\ 8-k \end{array} \left\{ \begin{array}{l} \mathcal{X}^i \\ \overline{* \mathcal{X}}_{-i} \end{array} \right. \begin{array}{l} 1 \\ 8-k \end{array} \left\{ \begin{array}{l} \text{MM}_e \\ \text{MM}_m \end{array} \right. \\
& {}_{9-k}^0 \text{rat} \left\{ \begin{array}{l} C_j - C_i \\ 3C_0 - C_{K \lrcorner ij} - 2C_j \end{array} \right. \begin{array}{l} 0 \\ 9-k \end{array} \left\{ \begin{array}{l} \emptyset_i^j \\ \overline{* \emptyset}_{-i}^j \end{array} \right. \begin{array}{l} 0 \\ 9-k \end{array} \left\{ \begin{array}{l} \text{KK}_e \\ \text{KK}_m \end{array} \right.
\end{aligned}$$