

$$\begin{aligned}
& \int_{d\vartheta}^{\mathbb{T}} \mathbf{e}^{\overline{\xi + \eta | \alpha \vartheta + \bar{\vartheta} \beta}} = \int_{d\vartheta}^{\mathbb{T}} \mathbf{e}^{\xi | \alpha \vartheta + \vartheta \beta | \eta} = \int_{d\vartheta}^{\mathbb{T}} \mathbf{e}^{\xi | \alpha \vartheta} \mathbf{e}^{\vartheta \beta | \eta} \\
&= \int_{d\vartheta}^{\mathbb{T}} \sum_i^{\mathbb{N}} \frac{\overline{\xi | \alpha \vartheta}^i}{i!} \sum_j^{\mathbb{N}} \frac{\overline{\vartheta \beta | \eta}^j}{j!} = \sum_i^{\mathbb{N}} \frac{\overline{\xi | \alpha}^i}{i!} \sum_j^{\mathbb{N}} \frac{\overline{\beta | \eta}^j}{j!} \int_{d\vartheta}^{\mathbb{T}} \vartheta^i \vartheta^j \\
&= \sum_m^{\mathbb{N}} \frac{\overline{\xi | \alpha}^m}{m!} \frac{\overline{\beta | \eta}^m}{m!} = \sum_m^{\mathbb{N}} \frac{\xi \eta^* \alpha \beta^*}{(m!)^2} = \sum_m^{\mathbb{N}} \frac{1}{m!} \xi \eta^* \mathcal{E}_{\alpha \beta^*}^m = \xi \eta^* \left[1 \right]_{\alpha \beta^*}
\end{aligned}$$