$$x^{-1} = \frac{{}^{x}\mathrm{grad}\Delta}{{}^{x}\Delta}$$

$$\frac{{}^{x}\overbrace{\partial_u \Delta}}{{}^{x}\Delta} = \frac{u|\,{}^{x}\mathrm{grad}\Delta}{{}^{x}\Delta} = u|\,x^{-1}$$

$${}^{x}\overbrace{\partial_u \partial_v \Delta} = {}^{x}\Delta\,\partial_v\,\underbrace{u|x^{-1}} + \underbrace{u|x^{-1}}\,\partial_v\,{}^{x}\Delta = -\,{}^{x}\Delta\,\partial_v\,\underbrace{u|vP_x^{-1}} + \underbrace{u|x^{-1}}\,\underbrace{v|x^{-1}}\,{}^{x}\Delta$$

$$\frac{{}^{x}\overbrace{\partial_u \partial_v \Delta}}{{}^{x}\Delta} = -\underbrace{u|vP_x^{-1}} + \underbrace{u|x^{-1}}\,\underbrace{v|x^{-1}}$$