

$$g_{mn} = \frac{g_{\mu\nu} + A_{\mu}^i g_{ij} A_{\nu}^j}{g_{ij} A_{\nu}^j} \Big| \frac{g_{ij} A_{\mu}^j}{g_{ij}}$$

$${}^{xy}g_{mn} = \frac{{}^xg_{\mu\nu} + {}^x\mathcal{X}_{\mu}^i {}^yg_{ij} {}^x\mathcal{X}_{\nu}^j}{{}^xg_{ij} {}^y\mathcal{X}_{\nu}^j} \Big| \frac{{}^yg_{ij} {}^x\mathcal{X}_{\mu}^j}{{}^yg_{ij}}$$

$${}^{xy}g_{mn} = \frac{{}^xg_{\mu\nu} {}^yh + {}^x\mathcal{X}_{\mu}^i {}^yg_{ij} {}^x\mathcal{X}_{\nu}^j}{{}^xg_{ij} {}^y\mathcal{X}_{\nu}^j} \Big| \frac{{}^yg_{ij} {}^x\mathcal{X}_{\mu}^j}{{}^x\phi^y g_{ij}}$$