

$$z:g \in D \times G \xrightarrow{\text{mult}} C \ni {}^z C_g$$

$${}^z C_{g'g} = {}^z C_g {}^{zg} C_{g'}$$

GCD

$${}^{zg} C_{wg} = {}^w C_g^* {}^z C_w {}^z C_g \in C^{\mathbb{C}}$$

$$\begin{array}{ccc}
 C^\chi \mathbf{z} D^\nu & \xrightarrow[{}^z D_w^\nu]{{}^z C_w^\chi} & C^\chi \mathbf{z} D^\nu \\
 \uparrow {}^z C_g^\chi \quad {}^z D_g^\nu & & \downarrow {}^w C_g^* \quad {}^w D_g^\nu \\
 C^\chi \mathbf{z} D^\nu & \xrightarrow[{}^{zg} D_{wg}^\nu]{{}^{zg} C_{wg}^\chi} & C^\chi \mathbf{z} D^\nu
 \end{array}$$

$$\text{LHS} = {}_C^{\circ g^*} I {}_C^{\circ g} {}_{zg} = \underbrace{{}_C^{\circ k g^*}}_z I \underbrace{{}_C^{\circ k g}}_z {}_{zg} = \overbrace{{}_C^{\circ k} {}_C^{\circ g} {}_C^{\circ z g}}^* I {}_C^{\circ k} {}_C^{\circ g} {}_C^{\circ z g} = {}_C^{\circ z g^*} {}_C^{\circ g^*} {}_C^{\circ k} I {}_C^{\circ k} {}_C^{\circ g} {}_C^{\circ z g} = {}_C^{\circ z g^*} {}_C^{\circ g^*} I {}_C^{\circ g} {}_C^{\circ z g} = \text{RHS}$$