

$$D_{\frac{2}{\omega}} C^X \xleftarrow[\text{metrep}]{C^X} G \times D_{\frac{2}{\omega}} C^X$$

$${}^z \overline{C_g^X \mathfrak{q}} = {}^z C_g^X {}^{zg} \mathfrak{q}$$

$$C_g^X \overline{C_{g'}^X \mathfrak{q}} = C_{gg'}^X \mathfrak{q}$$

$${}^z \text{LHS} = {}^z C_g^X {}^{zg} \overline{C_{g'}^X \mathfrak{q}} = {}^z C_g^X {}^{zg} C_{g'}^X {}^{zg, g'} \mathfrak{q} = {}^z C_{gg'}^X {}^{zg, g'} \mathfrak{q} = {}^z \text{RHS}$$

$$\overline{C_g^X \mathfrak{q}} \times \overline{C_g^X \mathfrak{q}} = \mathfrak{q} \times_{\chi} \mathfrak{q}$$

$$\begin{aligned} \overline{g_{\nu}^X \mathfrak{q}} \times \overline{g_{\nu}^X \mathfrak{q}} &= \int_{dz}^D \underbrace{{}^o \mathfrak{g}_z} \underbrace{{}^z g_{\nu}^X \mathfrak{q}} \times \underbrace{I_C} \underbrace{{}^o \mathfrak{g}_z} \underbrace{{}^z g_{\nu}^X \mathfrak{q}} = \int_{dz}^D \underbrace{{}^z g^{\nu} \quad {}^z g^{\nu}}_T \underbrace{{}^o \mathfrak{g}_z} \underbrace{{}^z g^z \mathfrak{q}} \times \underbrace{I_C} \underbrace{{}^o \mathfrak{g}_z} \underbrace{{}^z g^z \mathfrak{q}} \\ &= \int_{dz}^D \underbrace{{}^o \mathfrak{g}_{zg}} \underbrace{{}^z g \mathfrak{q}} \times \underbrace{I_C} \underbrace{{}^o \mathfrak{g}_{zg}} \underbrace{{}^z g \mathfrak{q}} = \int_{dz}^D \underbrace{{}^z \Delta_z^{\nu}} \underbrace{{}^o \mathfrak{g}_z} \underbrace{{}^z \mathfrak{q}} \times \underbrace{I_C} \underbrace{{}^o \mathfrak{g}_z} \underbrace{{}^z \mathfrak{q}} = \text{RHS} \end{aligned}$$