$$\mathbb{N} = \frac{n \in \mathbb{Z}}{n \geqslant 0}$$

$$\left(A_n\right)_{n \,\geqslant\, 0} \text{ sequence}$$

$$A\left(x\right) = \sum_{n \,\geqslant\, 0} A_n \, x^n \text{ formal power series}$$

$$A_n = \frac{1}{n!} \frac{d^n}{\left(dx\right)^n} A\left(x\right) \,\Big|_{x \,=\, 0}$$

$$A_n = 1 \Longrightarrow \text{ geom series } \frac{1}{1-x} = \sum_{n \,\geqslant\, 0} x^n$$

$$\alpha\left(\alpha-1\right)\left(\alpha-2\right)\cdots\left(\alpha-n+1\right) = {}_n(\alpha) \text{ down factorial n factors}$$

$$\alpha\left(\alpha+1\right)\left(\alpha+2\right)\cdots\left(\alpha+n-1\right) = \left(\alpha\right)_n \text{ up factorial n factors}$$

$$n! = \left(1\right)_n = {}_n(n)$$

$$\begin{bmatrix}\alpha\\ n\end{bmatrix} = \frac{\alpha\left(\alpha-1\right)\left(\alpha-2\right)\cdots\left(\alpha-n+1\right)}{n!} = \frac{{}_n(\alpha)}{n!} = \frac{{}_n(\alpha)}{{}_n(n)}$$

$$\text{binomial series } \left(1+x\right)^{\alpha} = \sum_{n \,\geqslant\, 0} \begin{bmatrix}\alpha\\ n\end{bmatrix} x^n$$

$$\left(-1\right)^n {}_n(-\alpha) = \left(\alpha\right)_n$$

$$\text{LHS } = \left(-1\right)^n \left(-\alpha\right)\left(-\alpha-1\right)\left(-\alpha-2\right)\cdots\left(-\alpha-n+1\right) = \alpha\left(\alpha+1\right)\left(\alpha+2\right)\cdots\left(\alpha+n-1\right) = \text{ RHS}$$

$$\left(1-x\right)^{-\alpha} = \sum_{n \,\geqslant\, 0} \frac{\left(\alpha\right)_n}{n!} x^n$$

$$\text{LHS } = \sum_{n \,\geqslant\, 0} \frac{{}_n(-\alpha)}{n!} \left(-x\right)^n = \sum_{n \,\geqslant\, 0} \left(-1\right)^n \frac{{}_n(-\alpha)}{n!} x^n = \text{ RHS}$$