

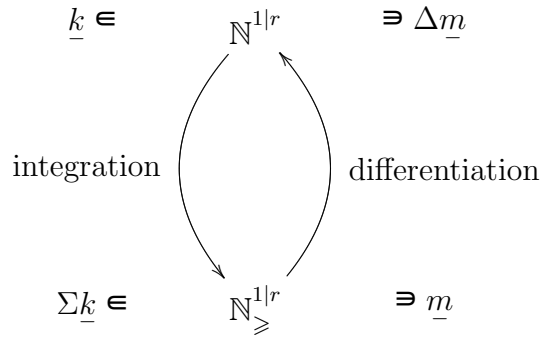
$M \subset \mathbb{N} \supset N$  zush = number intervals

$M = 1|r$  number interval

$$M^N = \{N \xrightarrow{\mathcal{V}} M\}$$

$$\mathbb{N}^{1|r} = \frac{\underline{k} = k_1 \cdots k_r \text{ r-tuples}}{k_i \geq 0}$$

multi-subset :  $x_1^{k_1} \cdots x_n^{k_n}$



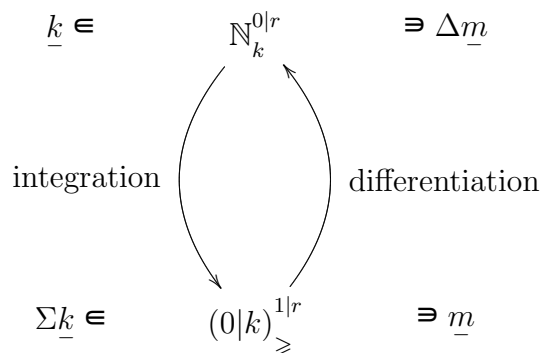
$$k_1 \cdots k_r \geq 0$$

$$m_i = k_i + \cdots + k_r \text{ integration}$$

$$m_1 \geq m_2 \geq \cdots \geq m_r \geq 0 = m_{r+}$$

$$k_i = m_i - m_{i+} \text{ differentiation}$$

$$\mathbb{N}_k^{0|r} = \frac{k_0 \cdots k_r \geq 0}{k_0 + \cdots + k_r = k} \text{ restricted tuples}$$



$$k_0 \cdots k_r \geq 0: \quad k_0 + \cdots + k_r = k$$

$$1 \leq i \leq r: \quad m_i = k_i + \cdots + k_r$$

$$\Rightarrow m_1 = k_1 + \cdots + k_r \leq k_0 + k_1 + \cdots + k_r = k$$

$$k \geq m_1 \geq m_2 \geq \cdots \geq m_r \geq 0 = m_{r+}$$

$$1 \leq i \leq r: \quad k_i = m_i - m_{i+}: \quad k_0 = k - m_1 \geq 0$$

$$k_0 + k_1 + \cdots + k_r = (k - m_1) + (m_1 - m_2) + \cdots + (m_{r-} - m_r) + m_r = k$$

subset  $0 \leq k_i \leq 1$

$$\# \frac{k_0 \cdots k_r \geq 0}{k_0 + \cdots + k_r = k} = \begin{bmatrix} k+r \\ r \end{bmatrix} = \begin{bmatrix} k+r \\ k \end{bmatrix}$$

bijection :  $k_0 () k_1 () \cdots () k_{r-} () k_r$ :  $r$  gaps in  $k+r$  set