$$S = \{ x_1 \cdot\cdot x_n \} \text{ distinct elements}$$

$$\sharp S = n$$

$$\text{subset } T \subset S$$

$$\varnothing \subset S \supset S$$

$$(S) = 2^S = \text{ all subsets}$$

$$\text{power recursiv } x^0 = 1: \quad x^{n+1} = x \cdot x^n$$

$$\sharp \mathcal{P}(S) = 2^n$$

recursiv

$$S = \{ x_0 \cdot\cdot x_n \}$$

$$(S) \asymp \left(S \llcorner x_0\right) \times 2 = \left(S \llcorner x_0\right) \ \dot{\cup} \ \left(S \llcorner x_0\right) \text{ disj union}$$

$$T \subset \{ x_0 \cdot\cdot x_n \}$$

$$\text{if } x_0 \in T \Longrightarrow T \llcorner x_0 \subset \{ x_1 \cdot\cdot x_n \} \Longrightarrow 2^n \text{ subsets}$$

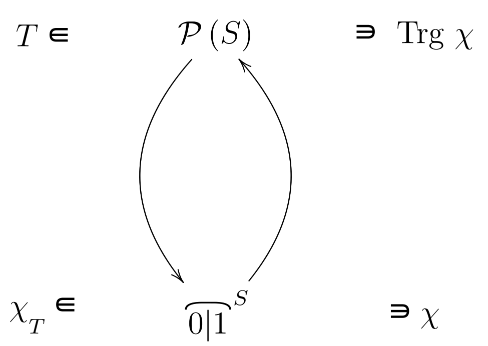
$$\text{if } x_0 \notin T \Longrightarrow T \subset \{ x_1 \cdot\cdot x_n \} \Longrightarrow 2^n \text{ subsets}$$

$$\Longrightarrow (S) = 2^n + 2^n = 2^{n+1} = 2 \cdot 2^n$$

bijektiv

$$\overbrace{0|1}^{1|n} = \{1|n \xrightarrow{\text{?}} 0|1\} = \frac{\underline{k} = k_1 \cdots k_n \text{ n-tuples}}{k_i \in 0|1}$$

$$\sharp \overbrace{0|1}^{S} = 2 \cdot 2 \cdots 2 = 2^n$$



$$\chi_T(i) = \begin{cases} 1 & i \in T \\ 0 & i \notin T \end{cases}$$

$$\text{Trg } \chi = \chi^{-1}(1)$$

$$\sum_{n \geqslant 0} 2^n x^n = \sum_{n \geqslant 0} (2x)^n = \frac{1}{1-2x}$$

$$0 \leqslant m \leqslant n: \quad \sharp T = m: \quad \sharp\varnothing = 0: \quad \sharp S = n$$

$$\widehat{0|1}^{1|n}_{m} = \text{ all subsets with m elements}$$

$$\begin{bmatrix} n \\ m \end{bmatrix} = \frac{n!}{m!\,(n-m)!} \text{ binomial}$$

$$\text{Pascal binomial recursion } \begin{bmatrix} n \\ 0 \end{bmatrix} = 1: \quad 1 \leqslant m \leqslant n: \quad \begin{bmatrix} n+ \\ m \end{bmatrix} = \begin{bmatrix} n \\ m- \end{bmatrix} + \begin{bmatrix} n \\ m \end{bmatrix}$$

$$\overbrace{0|1}_{m}^{1|n} = \begin{bmatrix} n \\ m \end{bmatrix}$$

$$0 = m = n \text{ klar}$$

$$\text{recursiv ind } n \geqslant 0$$

$$S = \{ x_1 \cdot\cdot x_n \}$$

$$S \cup x_0 = \{ x_0 \cdot\cdot x_n \}$$

$$T \in \overbrace{0|1}_{m}^{0|n}$$

$$\text{if } x_0 \in T \Rightarrow m \geqslant 1: \quad T \llcorner x_0 \in \overbrace{0|1}_{m-}^{1|n} \underset{\text{ind}}{\Longrightarrow} \begin{bmatrix} n \\ m- \end{bmatrix} \text{ subsets}$$

$$\text{if } x_0 \notin T \Rightarrow T \in \overbrace{0|1}_{m}^{1|n} \underset{\text{ind}}{\Longrightarrow} \begin{bmatrix} n \\ m \end{bmatrix} \text{ subsets}$$

$$\Longrightarrow \sharp \overbrace{0|1}_{m}^{0|n} = \begin{bmatrix} n \\ m- \end{bmatrix} + \begin{bmatrix} n \\ m \end{bmatrix} = \begin{bmatrix} n+1 \\ m \end{bmatrix}$$

$$\text{laufindex } m: \quad \sum_m^{0|n} \begin{bmatrix} n \\ m \end{bmatrix} x^m = (1+x)^n$$

$$(1+x)^{n+} = (1+x)^n (1+x) \underset{\text{ind}}{=} (1+x) \sum_m^{0|n} \begin{bmatrix} n \\ m \end{bmatrix} x^m = \sum_m^{0|n} \begin{bmatrix} n \\ m \end{bmatrix} x^m + x \sum_k^{0|n} \begin{bmatrix} n \\ k \end{bmatrix} x^k$$

$$= 1 + \sum_m^{1|n} \begin{bmatrix} n \\ m \end{bmatrix} x^m + \sum_k^{0|n-} \begin{bmatrix} n \\ k \end{bmatrix} x^{k+} + x^{n+}$$

$$= 1 + \sum_m^{1|n} \begin{bmatrix} n \\ m \end{bmatrix} x^m + \sum_m^{1|n} \begin{bmatrix} n \\ m- \end{bmatrix} x^m + x^{n+} = 1 + \sum_m^{1|n} \left(\begin{bmatrix} n \\ m \end{bmatrix} + \begin{bmatrix} n \\ m- \end{bmatrix} \right) x^m + x^{n+} = \sum_m^{0|n+} \begin{bmatrix} n+ \\ m \end{bmatrix} x^m$$

$$\left(\frac{d}{dx} \right)^m \frac{1}{1-x} = \frac{m!}{(1-x)^{m+}}$$

$$\text{laufindex n :} \quad \frac{x^m}{(1-x)^{m+}} = \sum_{n \geqslant m} \begin{bmatrix} n \\ m \end{bmatrix} x^n$$

$$\frac{m!}{(1-x)^{m+}} = \left(\frac{d}{dx}\right)^m \frac{1}{1-x} = \left(\frac{d}{dx}\right)^m \sum_{n \geqslant 0} x^n = \sum_{n \geqslant m} \left(\frac{d}{dx}\right)^m x^n = \sum_{n \geqslant m} {}_m(n)\, x^{n-m} = x^{-m} \sum_{n \geqslant m} {}_m(n)\, x^n$$

$$\frac{x^m}{(1-x)^{m+}} = \sum_{n \geqslant m} \frac{{}_m(n)}{m!} x^n = \text{ RHS}$$