$$n_1 \cdots n_\ell \cdots \in \underbrace{\mathbb{N}+1}_{} \bigstar \cdots \bigstar \underbrace{\mathbb{N}+1}_{} \cdots \xrightarrow{\Sigma} \mathbb{N} \ni n_1 + \cdots + n_\ell$$

$$P_0 = 1$$

$$P_n = \sharp \frac{n_1 \cdots n_\ell \in \overbrace{\mathbb{N}+1} \bigstar \cdots \bigstar \overbrace{\mathbb{N}+1}}{n_1 + \cdots + n_\ell = n}$$

partition exactly ℓ parts