

$$\ell + 1 \text{ cycle } \gamma = \overline{i_0 \cdots i_\ell}$$

$$\text{sgn} \gamma = (-1)^\ell$$

$$\pi = \gamma^1 | \cdots | \gamma^k \text{ disjoint cycles } \gamma^j \text{ length } \ell_j +$$

$$\text{sgn} \pi = \prod_j \text{sgn} \gamma^j = \prod_j (-1)^{\ell_j}$$

$$2 \text{ cycle} = \text{transposition } \tau = \overline{ij}$$

$$\tau^2 = \text{id} \neq \tau$$

$$\overline{i_0 \cdots i_\ell} = \overline{i_0 i_1} \overline{i_0 i_2} \cdots \overline{i_0 i_\ell}$$

$\ell + 1$ cycle = product of ℓ transpositions not disjoint

$$0 < i < \ell \Rightarrow \begin{cases} \text{LHS } i_h \mapsto i_{h+} \\ \text{RHS } i_h \mapsto i_0 \mapsto i_{h+} \end{cases}$$

$$\pi = \tau_1 \cdots \tau_m \text{ transposition} \Rightarrow \text{sgn}^\pi = (-1)^m$$

$$h \geq 0 \leq k: \{i_1 \cdots i_h i\} \cap \{j_1 \cdots j_k j\} = \emptyset \Rightarrow \overline{i_1 \cdots i_h i} \overline{j_1 \cdots j_k j} = \overline{i j} \overline{i_1 \cdots i_h i j_1 \cdots j_k j}$$

$$\text{sgn}^{\tau\pi} = -\text{sgn}^\pi$$

$$\tau = \overline{i j}: i \neq j$$

$$\text{if } i:j \in \text{ same cycle} \Rightarrow \pi = \overline{i_1 \cdots i_h i j_1 \cdots j_k j} | \alpha \Rightarrow \text{sgn}^\pi = (-1)^{1+h+k} \text{sgn}^\alpha$$

$$\tau\pi = \overline{i j} \overline{i_1 \cdots i_h i j_1 \cdots j_k j} | \alpha = \overline{i_1 \cdots i_h i} \overline{j_1 \cdots j_k j} | \alpha \Rightarrow \text{sgn}^{\tau\pi} = (-1)^{h+k} \text{sgn}^\alpha = -\text{sgn}^\pi$$

$$\text{if } i:j \in \text{ different cycles} \Rightarrow \pi = \overline{i_1 \cdots i_h i} \overline{j_1 \cdots j_k j} | \beta \Rightarrow \text{sgn}^\pi = (-1)^{h+k} \text{sgn}^\beta$$

$$\tau\pi = \tau | \overline{i_1 \cdots i_h i} \overline{j_1 \cdots j_k j} | \beta = \tau | \overline{i j} \overline{i_1 \cdots i_h i j_1 \cdots j_k j} | \beta = \overline{i_1 \cdots i_h i j_1 \cdots j_k j} | \beta$$

$$\Rightarrow \text{sgn}^{\tau\pi} = (-1)^{1+h+k} \text{sgn}^\beta = -\text{sgn}^\pi$$

$$\mathbb{C} | n \xrightarrow[\text{hom}]{\text{sgn}} \pm 1$$

$$\text{sgn}^{\sigma\pi} = \text{sgn}^\sigma \text{sgn}^\pi$$

$$\sigma = \tau_1 \cdots \tau_m: \pi = \tau^1 \cdots \tau^n \Rightarrow \sigma\pi = \tau_1 \cdots \tau_m \tau^1 \cdots \tau^n$$

$$\Rightarrow \text{LHS} = (-1)^{m+n} = (-1)^m (-1)^n = \text{RHS}$$