$$ {}_{m}\overset{-1}{\prec}{}^{n} \underset{\text{Moe}}{=} \begin{cases} \overset{k}{-1} & n/m = p_{1}\cdots p_{k} \text{ simple} \\ 0 & n/m \text{ not simple} \end{cases} $$

$$ \begin{cases} m = \prod_{p}^{\mathbb{P}} p^{m_p} \\ n = \prod_{p}^{\mathbb{P}} p^{n_p} \end{cases} $$

$$ m \prec n \Longrightarrow \begin{cases} \bigwedge_{p}^{\mathbb{P}} n_p \geqslant m_p \\ \dfrac{n}{m} = \prod_{p}^{\mathbb{P}} p^{n_p - m_p} \end{cases} $$

$$ {}_{m}\overset{-1}{\prec}{}^{n} = {}_{m.}\overset{-1}{\prec}{}^{n.} = \prod_{p}^{\mathbb{P}} {}_{m_p}\overset{-1}{\prec}{}^{n_p} = \prod_{p}^{\mathbb{P}} \begin{cases} 1 & m_p = n_p \\ -1 & n_p - m_p = 1 \\ 0 & n_p - m_p > 1 \end{cases} $$

$$ \frac{n}{m} \text{ not simple } \Longrightarrow \bigvee_{p}^{\mathbb{P}} n_p - m_p > 1 \Longrightarrow {}_{m_p}\overset{-1}{\prec}{}^{n_p} = 0 $$

$$ \text{simple } \frac{n}{m} = p_{1}\cdots p_{k} \Longrightarrow \bigwedge_{1 \leqslant j \leqslant k} n_{p_j} = m_{p_j} + 1 \Longrightarrow {}_{m_{p_j}}\overset{-1}{\prec}{}^{n_{p_j}} = -1 \Longrightarrow {}_{m.}\overset{-1}{\prec}{}^{n.} = \overset{k}{-1} $$

$$ {}_{1}\overset{-1}{\prec}{}^{n} \underset{\text{Moeb}}{\overset{\underline{\text{red}}}{=}} \begin{cases} \overset{k}{-1} & n = p_{1}\cdots p_{k} \text{ simple} \\ 0 & n \text{ not simple} \end{cases} $$