



$${}^r \mathbb{K}_s^\times = {}^r_C \mathbb{K}_r \cap {}^r_C \mathbb{K}_{r+s} = {}^r_U \mathbb{K}_r \cap {}^r_U \mathbb{K}_{r+s}$$

$$\mathbb{K}_r \Gamma = \mathbb{K}_r \mathbf{f} \Leftrightarrow \bigvee \Gamma \in {}^r \mathbb{K}_r^C : \mathbf{f} = \Gamma \Gamma$$

$$\Leftarrow : \mathbb{K}_r \mathbf{f} = \mathbb{K}_r \overline{\Gamma \Gamma} = \overline{\mathbb{K}_r \Gamma \Gamma} = \mathbb{K}_r \Gamma$$

$$\Rightarrow : \mathbb{K}_r \Gamma = \mathbb{K}_r \mathbf{f} \Rightarrow \bigvee {}^i \Gamma_j \in \mathbb{K} : \text{rows } {}^i \mathbf{f} = {}^i \Gamma_j {}^j \Gamma = {}^i \Gamma \Gamma : {}^i \Gamma \in {}^1 \mathbb{K}_r$$

$$\Rightarrow \Gamma = \begin{bmatrix} 1 \\ \Gamma \\ r \end{bmatrix} \in {}^r \mathbb{K}_r : \mathbf{f} = \Gamma \Gamma : \text{ analog } \Gamma = \mathbf{f} \mathbf{f}$$

$$\overline{\Gamma \mathbf{f}} \mathbf{f} = \overline{\Gamma \mathbf{f} \mathbf{f}} = \overline{\Gamma \Gamma} = \mathbf{f} \xrightarrow{\max} \Gamma \mathbf{f} = 1_r = \mathbf{f} \Gamma \Rightarrow \Gamma \in {}^r_C \mathbb{K}_r$$