$$< \mathbb{1}_- > \xrightarrow{\iota} \mathbb{1} \xrightarrow{\jmath} \mathbb{1} + < \mathbb{1}_- >$$

$$\daleth = \left({}_k\daleth^\ell \right) \in {}_p\mathbb{1}^q$$

$$\underset{\sim}{\daleth} := \left({}_k\underset{\sim}{\daleth}^\ell \right) \in {}_p\left(\mathbb{1} + < \mathbb{1}_- > \right)^q$$

$$\daleth \text{ inv } \Leftrightarrow \underset{\sim}{\daleth} \text{ inv}$$

$$\Rightarrow : \daleth \text{ inv } \Rightarrow \underset{\sim}{\daleth} = \diagdown\daleth \text{ inv}$$

$$\Leftarrow : \underset{\sim}{\daleth} \text{ inv } \Rightarrow \bigvee_{\acute{\daleth}} \underbrace{\daleth\acute{\daleth}}_{\sim} = \underset{\sim}{\daleth}\,\underset{\sim}{\acute{\daleth}} = I = \underset{\sim}{\acute{\daleth}}\,\underset{\sim}{\daleth} = \underbrace{\acute{\daleth}\daleth}_{\sim}$$

$$\gimel := I - \daleth\acute{\daleth} \Rightarrow \underset{\sim}{\gimel} = 0 \Rightarrow {}_k\underset{\sim}{\gimel}^\ell = 0 \Rightarrow {}_k\underset{\sim}{\overset{n+1}{\gimel}}{}^\ell = 0 \Rightarrow \overset{n+1}{\gimel} = 0$$

$$\daleth\acute{\daleth}\underbrace{I + \gimel + \cdots + \overset{n}{\gimel}} = \underbrace{I - \gimel}\underbrace{I + \gimel + \cdots + \overset{n}{\gimel}} = I - \overset{n+1}{\gimel} = 0 \Rightarrow \acute{\daleth}\underbrace{I + \gimel + \cdots + \overset{n}{\gimel}} \text{ right inv}$$

$$\beth := I - \acute{\daleth}\daleth \Rightarrow \underset{\sim}{\beth} = 0 \Rightarrow {}_k\underset{\sim}{\beth}^\ell = 0 \Rightarrow {}_k\underset{\sim}{\overset{n+1}{\beth}}{}^\ell = 0 \Rightarrow \overset{n+1}{\beth} = 0$$

$$\underbrace{I + \beth + \cdots + \overset{n}{\beth}}\acute{\daleth}\daleth = \underbrace{I + \beth + \cdots + \overset{n}{\beth}}\underbrace{I - \beth} = I - \overset{n+1}{\beth} = 0 \Rightarrow \underbrace{I + \beth + \cdots + \overset{n}{\beth}}\acute{\daleth} \text{ left inv}$$

$$\begin{array}{c|c} \daleth & \digamma \\ \hline \eth & \beth \end{array} = \begin{array}{c|c} {}_i\daleth^j & {}_i\digamma^n \\ \hline {}_m\eth^j & {}_m\beth^n \end{array} \text{ even}$$

$$\begin{array}{c|c} \daleth & \digamma \\ \hline \eth & \beth \end{array} \text{ inv } \Leftrightarrow \daleth \text{ inv } \beth \text{ inv}$$

$${}_i\digamma^n \in \mathbb{1}_- \ni {}_m\eth^j \Rightarrow {}_i\underset{\sim}{\digamma}^n = 0 = {}_m\underset{\sim}{\eth}^j \Rightarrow \underset{\sim}{\begin{array}{c|c} \daleth & \digamma \\ \hline \eth & \beth \end{array}} = \begin{array}{c|c} \underset{\sim}{\daleth} & 0 \\ \hline 0 & \underset{\sim}{\beth} \end{array}$$

$$\Rightarrow \begin{array}{c|c} \daleth & \digamma \\ \hline \eth & \beth \end{array} \text{ inv } \Leftrightarrow \underset{\sim}{\begin{array}{c|c} \daleth & \digamma \\ \hline \eth & \beth \end{array}} = \begin{array}{c|c} \underset{\sim}{\daleth} & 0 \\ \hline 0 & \underset{\sim}{\beth} \end{array} \text{ inv } \Leftrightarrow \underset{\sim}{\daleth} \text{ inv } \underset{\sim}{\beth} \Leftrightarrow \daleth \text{ inv } \beth$$

$$A \text{ inv } \; D \Rightarrow A - B D^{-1} C \text{ inv } \; D - C A^{-1} B$$

$$\begin{array}{c|c} A & B \\ \hline C & D \end{array}^{-1} = \begin{array}{c|c} \overbrace{A - B D^{-1} C}^{-1} & -A^{-1} B \overbrace{D - C A^{-1} B}^{-1} \\ \hline -D^{-1} C \overbrace{A - B D^{-1} C}^{-1} & \overbrace{D - C A^{-1} B}^{-1} \end{array}$$

$$\begin{cases} \begin{array}{c|c} A & B \\ \hline C & D \end{array} \begin{array}{c|c} I & 0 \\ \hline -D^{-1} C & I \end{array} = \begin{array}{c|c} A - B D^{-1} C & B \\ \hline 0 & D \end{array} \quad \Rightarrow A - B D^{-1} C \text{ inv} \\ \begin{array}{c|c} I & 0 \\ \hline -C A^{-1} & I \end{array} \begin{array}{c|c} A & B \\ \hline C & D \end{array} = \begin{array}{c|c} A & B \\ \hline 0 & D - C A^{-1} B \end{array} \quad \Rightarrow D - C A^{-1} B \text{ inv} \end{cases}$$

$$\begin{array}{c|c} A & B \\ \hline 0 & D \end{array}^{-1} = \begin{array}{c|c} A^{-1} & -A^{-1} B D^{-1} \\ \hline 0 & D^{-1} \end{array}$$

$$\begin{array}{c|c} A & B \\ \hline C & D \end{array}^{-1} = \begin{array}{c|c} I & 0 \\ \hline -D^{-1} C & I \end{array} \begin{array}{c|c} A - B D^{-1} C & B \\ \hline 0 & D \end{array}^{-1} = \begin{array}{c|c} I & 0 \\ \hline -D^{-1} C & I \end{array} \begin{array}{c|c} \overbrace{A - B D^{-1} C}^{-1} & -\overbrace{A - B D^{-1} C}^{-1} B D^{-1} \\ \hline 0 & D^{-1} \end{array}$$

$$= \begin{array}{c|c} \overbrace{A - B D^{-1} C}^{-1} & -\overbrace{A - B D^{-1} C}^{-1} B D^{-1} \\ \hline -D^{-1} C \overbrace{A - B D^{-1} C}^{-1} & D^{-1} C \overbrace{A - B D^{-1} C}^{-1} B D^{-1} + D^{-1} \end{array}$$

$$\begin{array}{c|c} A & B \\ \hline C & D \end{array}^{-1} = \begin{array}{c|c} A & B \\ \hline 0 & D - C A^{-1} B \end{array}^{-1} \begin{array}{c|c} I & 0 \\ \hline -C A^{-1} & I \end{array} = \begin{array}{c|c} A^{-1} & -A^{-1} B \overbrace{D - C A^{-1} B}^{-1} \\ \hline 0 & \overbrace{D - C A^{-1} B}^{-1} \end{array} \begin{array}{c|c} I & 0 \\ \hline -C A^{-1} & I \end{array}$$

$$= \begin{array}{c|c} A^{-1} + A^{-1} B \overbrace{D - C A^{-1} B}^{-1} C A^{-1} & -A^{-1} B \overbrace{D - C A^{-1} B}^{-1} \\ \hline -\overbrace{D - C A^{-1} B}^{-1} C A^{-1} & \overbrace{D - C A^{-1} B}^{-1} \end{array}$$

$$\overbrace{A - B D^{-1} C}^{-1} = A^{-1} + A^{-1} B \overbrace{D - C A^{-1} B}^{-1} C A^{-1}$$

$$\overbrace{A - B D^{-1} C}^{-1} B D^{-1} = A^{-1} B \overbrace{D - C A^{-1} B}^{-1}$$

$$D^{-1} C \overbrace{A - B D^{-1} C}^{-1} = \overbrace{D - C A^{-1} B}^{-1} C A^{-1}$$

$$D^{-1}C\left(A-BD^{-1}C\right)^{-1}BD^{-1}+D^{-1}=\left(D-CA^{-1}B\right)^{-1}$$

$$\mathrm{EX}\ \begin{array}{c|c} A-BD^{-1}C & 0 \\ \hline 0 & CA^{-1}B-D \end{array} = \begin{array}{c|c} A & B \\ \hline C & D \end{array}\ \begin{array}{c|c} A^{-1} & 0 \\ \hline 0 & -D^{-1} \end{array}\ \begin{array}{c|c} A & B \\ \hline C & D \end{array}$$

$$\begin{array}{c|c} A & B \\ \hline C & D \end{array} = \underset{2}{\begin{array}{c|c} I & BD^{-1} \\ \hline 0 & I \end{array}}\ \left(=C_{+}\right)\underset{2}{\begin{array}{c|c} A-BD^{-1}C & 0 \\ \hline 0 & D \end{array}}\ \left(=C_{0}\right)\underset{2}{\begin{array}{c|c} I & 0 \\ \hline D^{-1}C & I \end{array}}\ \left(=C_{-}\right)$$