

$${}^r\mathbb{K}_{r|s} = \frac{\Gamma \in {}^r\mathbb{K}_{r|s}}{\text{rg } \Gamma = r \max} = \frac{\Gamma \in {}^r\mathbb{K}_{r|s}}{\Gamma \Gamma^* \text{ inv}}$$

$${}^r\mathbb{U}_{r|s} = \frac{\Gamma \in {}^r\mathbb{K}_{r|s}}{\Gamma \Gamma^* = 1_r}$$

$$\text{cpt } {}^r\mathbb{U}_{r|s} \subset_{\text{abg}} {}^r\mathbb{C}_{r|s} \subset_{\text{off}} {}^r\mathbb{K}_{r|s}$$

$$\Gamma = \begin{bmatrix} {}^1\Gamma & {}^r\Gamma \end{bmatrix} \in {}^r\mathbb{K}_{r|s} \xrightarrow[r \text{ lin}]{F} \underbrace{{}^r\mathbb{K}}_{\triangle} \xrightarrow{r|s} {}^r\mathbb{K} \ni {}^1\Gamma \wedge {}^r\Gamma$$

$${}^1\Gamma \wedge {}^r\Gamma \begin{bmatrix} \Gamma_1 \\ \Gamma_r \end{bmatrix} = {}^1\Gamma \wedge {}^r\Gamma | \Gamma = \det \left({}^i\Gamma \Gamma_j \right) = \det \Gamma \Gamma$$

$${}^r\mathbb{C}_{r|s} \xrightarrow[\text{hol}]{F} \underbrace{{}^r\mathbb{K}}_{\triangle} \xrightarrow{r|s} {}^r\mathbb{K} \hookrightarrow 0$$

$$\Gamma = [\Gamma_1 \quad \Gamma_{r|s}] \text{ unit basis} \Rightarrow \Gamma_m = \Gamma \Gamma_m$$

$$\text{rg } \Gamma = r \Rightarrow \bigvee_{m_1 << m_r} [\Gamma_{m_1} \quad \Gamma_{m_r}] \in {}^r\mathbb{C}_{r|s}$$

$$\begin{aligned} {}^1\Gamma \wedge {}^r\Gamma \begin{bmatrix} \Gamma_{m_1} \\ \Gamma_{m_r} \end{bmatrix} &= \det \left({}^i\Gamma \Gamma_{m_j} \right) = \det \left({}^i\Gamma_{m_j} \right) \neq 0 \\ &\Rightarrow {}^1\Gamma \wedge {}^r\Gamma \neq 0 \end{aligned}$$

$${}^1\overline{\Gamma \Gamma} \wedge {}^r\overline{\Gamma \Gamma} = \det \Gamma \underbrace{{}^1\Gamma \wedge {}^r\Gamma}$$

$$\text{LHS } | \Gamma = \det \overline{\Gamma \Gamma} \Gamma = \det \Gamma \overline{\Gamma \Gamma} = \det \Gamma \det \Gamma \Gamma = \text{RHS } | \Gamma$$

$$\begin{array}{ccc}
{}^r\mathbb{K}_{r|s} & \xrightarrow{F} & \underbrace{{}^r\mathbb{K} \triangleleft {}^{r|s}\mathbb{K}}_{\simeq 0} \\
\pi \downarrow & & \downarrow \tilde{\pi} \\
{}^r\mathbb{K}_s^\times & \xrightarrow{\mathcal{F}} & \mathbb{P} \underbrace{{}^r\mathbb{K} \triangleleft {}^{r|s}\mathbb{K}}
\end{array}$$

$$\mathbb{K}_r \Gamma = \mathbb{K}_r \mathcal{Y} \Leftrightarrow \bigvee \Gamma \in {}^r\mathbb{K}_r: \mathcal{Y} = \Gamma \Gamma$$

$$\Rightarrow {}^1\mathcal{Y} \ddot{\wedge} {}^r\mathcal{Y} = \det \Gamma \underbrace{{}^1\Gamma \ddot{\wedge} {}^r\Gamma}_{\det \Gamma \neq 0} \Rightarrow [{}^1\mathcal{Y} \ddot{\wedge} {}^r\mathcal{Y}] = [{}^1\Gamma \ddot{\wedge} {}^r\Gamma]$$

\mathcal{F} injektiv

$$[{}^1\mathcal{Y} \ddot{\wedge} {}^r\mathcal{Y}] = [{}^1\Gamma \ddot{\wedge} {}^r\Gamma] \Rightarrow \bigvee_{\lambda \neq 0} {}^1\mathcal{Y} \ddot{\wedge} {}^r\mathcal{Y} = \lambda {}^1\Gamma \ddot{\wedge} {}^r\Gamma$$

$$\left[\begin{array}{cc} {}^1\Gamma_{m_1} & {}^r\Gamma_{m_r} \end{array} \right] \text{ frei} \Rightarrow {}^1\Gamma \ddot{\wedge} {}^r\Gamma \left[\begin{array}{c} \Gamma_{m_1} \\ \Gamma_{m_r} \end{array} \right] = \det \left({}^i\Gamma_{m_j} \right) \neq 0$$

$$\Rightarrow \det \left({}^i\mathcal{Y}_{m_j} \right) = {}^1\mathcal{Y} \ddot{\wedge} {}^r\mathcal{Y} \left[\begin{array}{c} \Gamma_{m_1} \\ \Gamma_{m_r} \end{array} \right] = \lambda {}^1\Gamma \ddot{\wedge} {}^r\Gamma \left[\begin{array}{c} \Gamma_{m_1} \\ \Gamma_{m_r} \end{array} \right] = \lambda \det \left({}^i\Gamma_{m_j} \right) \neq 0$$

$$\Rightarrow \left[\begin{array}{cc} {}^1\mathcal{Y}_{m_1} & {}^r\mathcal{Y}_{m_r} \end{array} \right] \text{ frei} \Rightarrow \bigvee \Gamma \in {}^r\mathbb{K}_r: \left[\begin{array}{cc} {}^1\mathcal{Y}_{m_1} & {}^r\mathcal{Y}_{m_r} \end{array} \right] = \Gamma \left[\begin{array}{cc} {}^1\Gamma_{m_1} & {}^r\Gamma_{m_r} \end{array} \right]$$

$$\mathbb{K}_r \mathcal{Y} = \mathbb{K}_r \left[\begin{array}{cc} {}^1\mathcal{Y}_{m_1} & {}^r\mathcal{Y}_{m_r} \end{array} \right] = \mathbb{K}_r \Gamma \left[\begin{array}{cc} {}^1\Gamma_{m_1} & {}^r\Gamma_{m_r} \end{array} \right] = \mathbb{K}_r \left[\begin{array}{cc} {}^1\Gamma_{m_1} & {}^r\Gamma_{m_r} \end{array} \right] = \mathbb{K}_r \Gamma$$