$$ {}^{r}_{\mathsf{C}}\mathbb{K}_{r|s} = \frac{\Gamma \in {}^{r}\mathbb{K}_{r|s}}{\mathrm{rg}\ \Gamma = r\ \max} = \frac{\Gamma \in {}^{r}\mathbb{K}_{r|s}}{\Gamma \overset{*}{\Gamma}\ \mathrm{inv}} $$

$$ {}^{r}_{\mathsf{U}}\mathbb{K}_{r|s} = \frac{\Gamma \in {}^{r}\mathbb{K}_{r|s}}{\Gamma \overset{*}{\Gamma} = 1_r} $$

$$ \mathrm{cpt}\ {}^{r}_{\mathsf{U}}\mathbb{K}_{r|s} \underset{\mathrm{abg}}{\subset} {}^{r}_{\mathsf{C}}\mathbb{K}_{r|s} \underset{\mathrm{off}}{\subset} {}^{r}\mathbb{K}_{r|s} $$

$$ \Gamma = \left[{}^{1}\Gamma \quad {}^{r}\Gamma\right] \in {}^{r}\mathbb{K}_{r|s} \xrightarrow[r\ \mathrm{lin}]{F} \overset{r}{\mathbb{K}} \measuredangle^{r|s}\mathbb{K} \ni {}^{1}\Gamma \check{\wedge} {}^{r}\Gamma $$

$$ {}^{1}\Gamma \check{\wedge} {}^{r}\Gamma \begin{bmatrix} \Gamma_1 \\ \Gamma_r \end{bmatrix} = {}^{1}\Gamma \check{\wedge} {}^{r}\Gamma | \Gamma = \det \left({}^{i}\Gamma \Gamma_j\right) = \det \Gamma \Gamma $$

$$ {}^{r}_{\mathsf{C}}\mathbb{K}_{r|s} \xrightarrow[\mathrm{hol}]{F} \underbrace{\overset{r}{\mathbb{K}} \measuredangle^{r|s}\mathbb{K}} \llcorner 0 $$

$$ \Gamma = \left[\Gamma_1 \quad \Gamma_{r|s}\right] \text{ unit basis } \Rightarrow \Gamma_m = \Gamma \Gamma_m $$

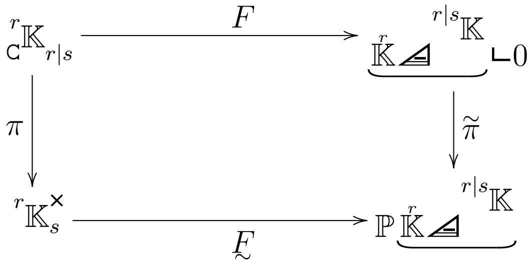
$$ \mathrm{rg}\ \Gamma = r \Rightarrow \bigvee_{m_1 << m_r} \left[\Gamma_{m_1} \quad \Gamma_{m_r}\right] \in {}^{r}_{\mathsf{C}}\mathbb{K}_r $$

$$ {}^{1}\Gamma \check{\wedge} {}^{r}\Gamma \begin{bmatrix} \Gamma_{m_1} \\ \Gamma_{m_r} \end{bmatrix} = \det \left({}^{i}\Gamma \Gamma_{m_j}\right) = \det \left({}^{i}\Gamma_{m_j}\right) \neq 0 $$

$$ \Rightarrow {}^{1}\Gamma \check{\wedge} {}^{r}\Gamma \neq 0 $$

$$ {}^{1}\overbrace{\Gamma \Gamma} \check{\wedge} {}^{r}\overbrace{\Gamma \Gamma} = \det \Gamma \underbrace{{}^{1}\Gamma \check{\wedge} {}^{r}\Gamma} $$

$$ \mathrm{LHS}\ |\Gamma = \det \underbrace{\Gamma \Gamma} \Gamma = \det \Gamma \underbrace{\Gamma \Gamma} = \det \Gamma \det \Gamma \Gamma = \mathrm{RHS}\ |\Gamma $$



$$\mathbb{K}_r\,\Gamma = \mathbb{K}_r\,Y \Leftrightarrow \bigvee \Gamma \in {}^{r}_{\mathsf{C}}\mathbb{K}_r: \quad Y = \Gamma\,\Gamma$$

$$\Longrightarrow {}^{1}Y \ddot{\wedge} {}^{r}Y = \det \Gamma \underbrace{{}^{1}\Gamma \ddot{\wedge} {}^{r}\Gamma} \underset{\det \Gamma \neq 0}{\Longrightarrow} [\,{}^{1}Y \ddot{\wedge} {}^{r}Y\,] = [\,{}^{1}\Gamma \ddot{\wedge} {}^{r}\Gamma\,]$$

$\underset{\sim}{F}$ injektiv

$$[\,{}^{1}Y \ddot{\wedge} {}^{r}Y\,] = [\,{}^{1}\Gamma \ddot{\wedge} {}^{r}\Gamma\,] \Rightarrow \bigvee_{\lambda \neq 0} {}^{1}Y \ddot{\wedge} {}^{r}Y = \lambda\, {}^{1}\Gamma \ddot{\wedge} {}^{r}\Gamma$$

$$\left[{}^{1}\Gamma_{m_1} \quad {}^{r}\Gamma_{m_r}\right] \text{ frei } \Longrightarrow {}^{1}\Gamma \ddot{\wedge} {}^{r}\Gamma \begin{bmatrix} \Gamma_{m_1} \\ \Gamma_{m_r} \end{bmatrix} = \det\left({}^{i}\Gamma_{m_j}\right) \neq 0$$

$$\Longrightarrow \det\left({}^{i}Y_{m_j}\right) = {}^{1}Y \ddot{\wedge} {}^{r}Y \begin{bmatrix} \Gamma_{m_1} \\ \Gamma_{m_r} \end{bmatrix} = \lambda\, {}^{1}\Gamma \ddot{\wedge} {}^{r}\Gamma \begin{bmatrix} \Gamma_{m_1} \\ \Gamma_{m_r} \end{bmatrix} = \lambda \det\left({}^{i}\Gamma_{m_j}\right) \neq 0$$

$$\Longrightarrow \left[{}^{1}Y_{m_1} \quad {}^{r}Y_{m_r}\right] \text{ frei } \Longrightarrow \bigvee \Gamma \in {}^{r}_{\mathsf{C}}\mathbb{K}_r: \quad \left[{}^{1}Y_{m_1} \quad {}^{r}Y_{m_r}\right] = \Gamma \left[{}^{1}\Gamma_{m_1} \quad {}^{r}\Gamma_{m_r}\right]$$

$$\mathbb{K}_r\,Y = \mathbb{K}_r \left[{}^{1}Y_{m_1} \quad {}^{r}Y_{m_r}\right] = \mathbb{K}_r\,\Gamma \left[{}^{1}\Gamma_{m_1} \quad {}^{r}\Gamma_{m_r}\right] = \mathbb{K}_r \left[{}^{1}\Gamma_{m_1} \quad {}^{r}\Gamma_{m_r}\right] = \mathbb{K}_r\,\Gamma$$