$$I \in \begin{bmatrix} r+s \\ r \end{bmatrix}$$

$${}^{r}_{\mathsf{C}}\mathbb{K}^{I}_{r+s} = \frac{\Gamma \in {}^{r}\mathbb{K}_{r+s}}{\Gamma_{I} \in {}^{r}\mathbb{K}_{I} \text{ inv}} \underset{\text{off}}{\subset} {}^{r}\mathbb{K}_{r+s}$$

$$\pi \text{ off } \Longrightarrow \mathbb{K}_{r}\, {}^{r}_{\mathsf{C}}\mathbb{K}^{I}_{r+s} \underset{\text{off}}{\subset} {}^{r}\mathbb{K}^{\times}_{s}$$

$${}^{r}_{\mathsf{C}}\mathbb{K}_{r+s} = \bigcup_{I} {}^{r}_{\mathsf{C}}\mathbb{K}^{I}_{r+s}$$

$$\pi \text{ surj } \Longrightarrow {}^{r}\mathbb{K}^{\times}_{s} = \mathbb{K}_{r}\, {}^{r}_{\mathsf{C}}\mathbb{K}_{r+s} = \bigcup_{I} \mathbb{K}_{r}\, {}^{r}_{\mathsf{C}}\mathbb{K}^{I}_{r+s}$$

$${}^{r}_{\mathsf{C}}\mathbb{K}^{I}_{r+s} = \frac{\underbrace{\Gamma | \nearrow}\, P_{I}}{\Gamma \in {}^{r}_{\mathsf{C}}\mathbb{K}_{r}}$$

$$\nearrow \in {}^{r}\mathbb{K}_{s} \xrightarrow[\text{bij}]{\Phi_{I}} \mathbb{K}_{r}\, {}^{r}_{\mathsf{C}}\mathbb{K}^{I}_{r+s} \ni \mathbb{K}_{r} \underbrace{1 | \nearrow}\, P_{I} \sqsubset \mathbb{K}_{r+s}$$

$$\begin{cases} \mathbb{K}_{r} \underbrace{\Gamma | \nearrow}\, P_{I} \\ \mathbb{K}_{r}\Gamma \end{cases} \in \mathbb{K}_{r}\, {}^{r}_{\mathsf{C}}\mathbb{K}^{I}_{r+s} \xrightarrow[\text{bij}]{\Phi_{I}^{-1}} {}^{r}\mathbb{K}_{s} \ni \begin{cases} \overset{-1}{\Gamma} \nearrow \\ \underbrace{\Gamma \overset{-1}{P}_{I}}_{r|}{}^{-1} \underbrace{\Gamma \overset{-1}{P}_{I}}_{|s} \end{cases}$$

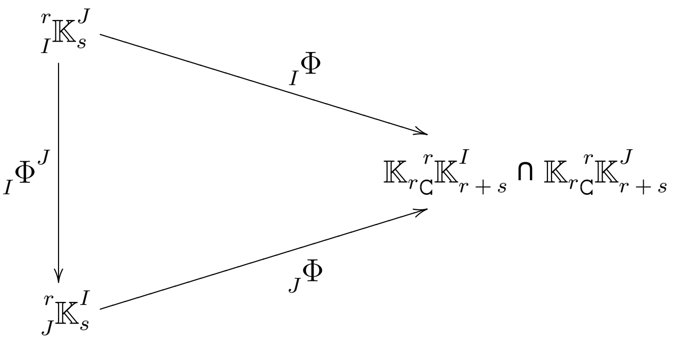
$$\Phi_{I}^{-1} \text{ well-def}$$

$$\searrow \underbrace{\Gamma | \nearrow}\, P_{I} = \underbrace{\searrow \Gamma | \searrow \nearrow}\, P_{I} \mapsto \overbrace{\searrow \Gamma}^{-1} \underbrace{\searrow \nearrow} = \overset{-1}{\Gamma}\, \overset{-1}{\searrow} \searrow \nearrow = \overset{-1}{\Gamma} \nearrow$$

$$\Phi_I^{-1} \circ \Phi_I = \text{ id } = \Phi_I \circ \Phi_I^{-1}$$

$$\Phi_I^{-1} \circ \Phi_I (\lrcorner) = \Phi_I^{-1} \left(\mathbb{K}_r \underbrace{1|\lrcorner} P_I\right) = 1^{-1} \lrcorner = \lrcorner$$

$$\Phi_I \circ \Phi_I^{-1} \left(\mathbb{K}_r \underbrace{\ulcorner|\lrcorner} P_I\right) = \Phi_I \left(\overset{-1}{\ulcorner} \lrcorner\right) = \mathbb{K}_r \underbrace{1|\overset{-1}{\ulcorner} \lrcorner} P_I = \mathbb{K}_r \overset{-1}{\ulcorner} \underbrace{\ulcorner|\lrcorner} P_I = \mathbb{K}_r \underbrace{\ulcorner|\lrcorner} P_I$$



$${}_I\Phi^J (\lrcorner) = {}_J\Phi^{-1} \circ {}_I\Phi (\lrcorner) = {}_J\Phi^{-1} \left(\mathbb{K}_r \underbrace{1|\lrcorner} P_I\right) = \underbrace{\overbrace{1|\lrcorner} P_I \overset{-1}{P}_J}_{r|}{}^{-1} \underbrace{\overbrace{1|\lrcorner} P_I \overset{-1}{P}_J}_{|s}$$