

$${}^{\mathbb{C}}_m q^m = q^m \triangleleft q^m = \left\{ q^m \xrightarrow[\text{bij lin}]{\mathcal{A}} q^m \right\} \ni \mathcal{A} = \begin{bmatrix} \mathcal{A} \\ 0 \\ \vdots \\ \mathcal{A} \end{bmatrix}_{m-1} \text{ free}$$

$$\# {}^{\mathbb{C}}_m q^m = \prod_i^m \underbrace{q^m - q^i}$$

$$\text{LHS} = \# \mathcal{A} = \underbrace{q^m - 1} \underbrace{q^m - q} \underbrace{q^m - q^2} \cdots \underbrace{q^m - q^{m-1}} = \text{RHS}$$