$$\det \begin{array}{c|c|c|c|c} a_0^1 & a_0^2 & \dots & a_0^n & a_0^{n+} \\ \hline 1 & a_1^2 & \dots & a_1^n & a_1^{n+} \\ \hline 0 & \ddots & \ddots & \vdots & \vdots \\ \hline 0 & 0 & \ddots & a_{n-}^n & a_{n-}^{n+} \\ \hline 0 & 0 & 0 & 1 & a_n^{n+} \end{array} = \sum_{\ell}^{0|n} (-1)^{n-\ell} \sum_{i_1 << i_\ell}^{1|n} a_0^{i_1}\, a_{i_1}^{i_2} \cdots a_{i_\ell}^{n+}$$

$$\det \begin{array}{c|c|c|c|c|c} a_0^1 & a_0^2 & \dots & a_0^{n-} & a_0^n & a_0^{n+} \\ \hline 1 & a_1^2 & \dots & a_1^{n-} & a_1^n & a_1^{n+} \\ \hline 0 & \ddots & \ddots & \vdots & \vdots & \vdots \\ \hline 0 & 0 & \ddots & a_{n-2}^{n-} & a_{n-2}^n & a_{n-2}^{n+} \\ \hline 0 & 0 & 0 & 1 & a_{n-}^n & a_{n-}^{n+} \\ \hline 0 & 0 & 0 & 0 & 1 & a_n^{n+} \end{array}$$

$$= \det \begin{array}{c|c|c|c|c} a_0^1 & a_0^2 & \dots & a_0^{n-} & a_0^n \\ \hline 1 & a_1^2 & \dots & a_1^{n-} & a_1^n \\ \hline 0 & \ddots & \ddots & \vdots & \vdots \\ \hline 0 & 0 & \ddots & a_{n-2}^{n-} & a_{n-2}^n \\ \hline 0 & 0 & 0 & 1 & a_{n-}^n \end{array} \; a_n^{n+} - \det \begin{array}{c|c|c|c|c} a_0^1 & a_0^2 & \dots & a_0^{n-} & a_0^{n+} \\ \hline 1 & a_1^2 & \dots & a_1^{n-} & a_1^{n+} \\ \hline 0 & \ddots & \ddots & \vdots & \vdots \\ \hline 0 & 0 & \ddots & a_{n-2}^{n-} & a_{n-2}^{n+} \\ \hline 0 & 0 & 0 & 1 & a_{n-}^{n+} \end{array}$$

$$= \sum_{\ell}^{1|n} (-1)^{(n-1)-(\ell-1)} \sum_{i_1 << i_{\ell-}}^{1|n-} a_0^{i_1}\, a_{i_1}^{i_2} \cdots a_{i_{\ell-}}^n\, a_n^{n+} - \sum_{\ell}^{0|n-} (-1)^{n-1-\ell} \sum_{i_1 << i_\ell}^{1|n-} a_0^{i_1}\, a_{i_1}^{i_2} \cdots a_{i_\ell}^{n+}$$

$$= \sum_{\ell}^{1|n} (-1)^{n-\ell} \sum_{i_1 << i_\ell = n}^{1|n} a_0^{i_1}\, a_{i_1}^{i_2} \cdots a_{i_\ell}^{n+} + \sum_{\ell}^{0|n-} (-1)^{n-\ell} \sum_{i_1 << i_\ell < n}^{1|n} a_0^{i_1}\, a_{i_1}^{i_2} \cdots a_{i_\ell}^{n+} = \text{ RHS}$$

$$\det \begin{array}{c|c|c} a_0^1 & a_0^2 & a_0^3 \\ \hline a_1^1 & a_1^2 & a_1^3 \\ \hline 0 & a_2^2 & a_2^3 \end{array} = \underbrace{a_0^1 a_1^2 a_2^3}_{j=2} + a_1^1\, a_2^2\, \underbrace{a_0^3}_{j=0} - a_1^1\, \underbrace{a_0^2 a_2^3}_{j=1} - \underbrace{a_0^1 a_1^3}_{j=1}\, a_2^3$$