

$${}^{\beta}\nabla = (d^{\beta}g_{\alpha})^{\alpha}g_{\beta} + {}^{\beta}g_{\alpha}{}^{\alpha}\nabla^{\alpha}g_{\beta}$$