$$ {}^{\alpha}\overbrace{d_{\partial \log h}\underbrace{\partial \log h}} \;=\; \bar{\partial}\,{}^{\alpha}\overbrace{\partial \log h} $$

$$ -\partial\, h_\alpha \;=\; h_\alpha\, \underbrace{\partial^\alpha h}\, h_\alpha $$

$$ \partial\overbrace{\underbrace{\partial^\alpha h} h_\alpha} \;=\; \overbrace{\partial\underbrace{\partial^\alpha h}}^{=0}\; h_\alpha - \underbrace{\partial^\alpha h} \wedge \underbrace{\partial h_\alpha} = -\underbrace{\partial^\alpha h} \wedge \underbrace{\partial h_\alpha} = \underbrace{\partial^\alpha h} \wedge h_\alpha\, \underbrace{\partial^\alpha h}\, h_\alpha $$

$$ \text{LHS} \;= d\,{}^{\alpha}\overbrace{\partial \log h} \;-\; {}^{\alpha}\overbrace{\partial \log h} \;\wedge\; {}^{\alpha}\overbrace{\partial \log h} \;= d\overbrace{\underbrace{\partial^\alpha h} h_\alpha} - \underbrace{\partial^\alpha h}\, h_\alpha \wedge \underbrace{\partial^\alpha h}\, h_\alpha $$

$$ = \bar{\partial}\overbrace{\underbrace{\partial^\alpha h} h_\alpha} + \partial\overbrace{\underbrace{\partial^\alpha h} h_\alpha} - \underbrace{\partial^\alpha h}\, h_\alpha \wedge \underbrace{\partial^\alpha h}\, h_\alpha = \bar{\partial}\overbrace{\underbrace{\partial^\alpha h} h_\alpha} = \;\text{RHS} $$