

$$0\text{-stellig } O = \underline{U} = \mathcal{F}_0$$

$$U \text{ Peano alg} \Leftrightarrow \begin{cases} U = \bar{O} = \bar{U} \text{ minimal} \\ U^n \xrightarrow[f_n]{\text{inj}} U \\ U = O \cup \bigcup_{f_n} f_n U^n \text{ disj union} \end{cases}$$

$$a \in U \Rightarrow \bigvee_{\text{eind}} a = F_n a_1 \cdots a_n$$

$$\text{Zustandsraum : } Y \ni \begin{cases} \tilde{o} \\ \tilde{f}_n \underbrace{a_1 : y_1 \cdots a_n : y_n} \end{cases}$$

$$\text{Funktion alg } U \times Y \ni \begin{cases} \tilde{o} = o | \tilde{o} \\ \tilde{f}_n \underbrace{a_1 | y_1 \cdots a_n | y_n} = \overbrace{f_n a_1 \cdots a_n} | \underbrace{\tilde{f}_n a_1 | y_1 \cdots a_n | y_n} \end{cases}$$

$$0\text{-stellig } \underline{U \times Y} = \frac{[o \ \tilde{o}]}{o \in O = \underline{U}}$$

$$U \text{ Peano alg} \xrightarrow[\text{Satz}]{\text{Rek}} \bigvee a \in U \xrightarrow{()^\sim} Y \ni \tilde{a}: \begin{cases} \tilde{\delta} = \delta \\ \overline{f_n a_1 \cdots a_n} = \tilde{f}_n \underbrace{[a_1 \cdots a_n]}_{\tilde{a}_1 \cdots \tilde{a}_n} \end{cases}$$

$$V = \frac{a \in U}{\substack{\text{eind} \\ \bigvee_{\tilde{a} \in Y} a: \tilde{a} \in \overline{U \times Y} \text{ ableitbar}}} \xrightarrow{()^\sim} Y$$

$$O = \underline{U} \subset V \underset{\text{abg}}{\subset} U$$

$$f_n \in \mathcal{F}_n: a_1 \cdots a_n \in V \xrightarrow[\text{Beh}]{\Rightarrow} f_n a_1 \cdots a_n \in V$$

$$\bigwedge_{1 \leq m \leq n} \bigvee_{\tilde{a}_m \in Y}^{\text{eind}} a_m: \tilde{a}_m \in \overline{U \times Y} \underset{\text{abg}}{\subset} U \times Y$$

$$\left[f_n a_1 \cdots a_n \quad \tilde{f}_n [a_1 \quad \tilde{a}_1] \cdots [a_n \quad \tilde{a}_n] \right] = \tilde{f}_n [a_1 \quad \tilde{a}_1] \cdots [a_n \quad \tilde{a}_n] \in \overline{U \times Y} \xrightarrow[\text{Ex}]{\Rightarrow} \overline{f_n a_1 \cdots a_n} = \tilde{f}_n [a_1 \quad \tilde{a}_1] \cdots [a_n \quad \tilde{a}_n]$$

$$\left[f_n a_1 \cdots a_n \quad y \right] \in \overline{U \times Y} \Rightarrow \bigvee_{\text{Abl}} \underbrace{b_1 | y_1} \cdots \underbrace{b_m | y_m} \underbrace{f_n a_1 \cdots a_n | y}$$

$$\Rightarrow \bigvee_{1 \leq i_1 < \cdots < i_k \leq m} \overbrace{f_n a_1 \cdots a_n | y} = \tilde{g}_k \overbrace{b_{i_1} | y_{i_1} \cdots b_{i_k} | y_{i_k}} = \overbrace{g_k b_{i_1} \cdots b_{i_k}} | \tilde{g}_k \overbrace{b_{i_1} | y_{i_1} \cdots b_{i_k} | y_{i_k}}$$

$$f_n a_1 \cdots a_n = b_{i_1} \cdots b_{i_k} \xrightarrow[\text{Pea}]{\Rightarrow} n = k: f_n = g_n: \bigwedge_{1 \leq m \leq n} a_m = b_{i_m}$$

$$b_{i_m} = a_m \in V: \left[a_m \quad y_{i_m} \right] \in \overline{U \times Y} \xrightarrow[\text{eind}]{\Rightarrow} y_{i_m} = \tilde{a}_m$$

$$\Rightarrow y = \tilde{g}_n \left[b_{i_1} \quad y_{i_1} \right] \cdots \left[b_{i_n} \quad y_{i_n} \right] = \tilde{f}_n [a_1 \quad \tilde{a}_1] \cdots [a_n \quad \tilde{a}_n] \Rightarrow \text{eind}$$

$$\xrightarrow[\text{Satz}]{\text{Ind}} U = \bar{O} = V$$