$$\overline{\sum_{k<\ell} n^{-k} P_n\left(\mathcal{L}_k\left(f \text{⧗} g\right) p\right)(0) - \underbrace{\mathcal{T}_f^n \mathcal{T}_g^n}\, p\,(0)} \leqslant C\, n^{-\ell}$$

$$F_k = \mathcal{L}_k\left(f \text{⧗} \underbrace{gp}\right) - \mathcal{L}_k\left(f \text{⧗} g\right) p$$

$$\mathcal{T}_f^n\, \mathcal{T}_{gp}^n 1 = \mathcal{T}_f^n\, \mathcal{T}_g^n\, p$$

$$\overline{\sum_{k<\ell} n^{-k} P_n\left(F_k\right)(0)} = \overline{\sum_{k<\ell} n^{-k} P_n\left(\mathcal{L}_k\left(f \text{⧗} \underbrace{gp}\right) 1\right)(0) - \sum_{k<\ell} n^{-k} P_n\left(\mathcal{L}_k\left(f \text{⧗} g\right) p\right)(0)}$$

$$\leqslant \overline{\sum_{k<\ell} n^{-k} P_n\left(\mathcal{L}_k\left(f \text{⧗} \underbrace{gp}\right) 1\right)(0) - \mathcal{T}_f^n \mathcal{T}_{gp}^n 1\,(0)} + \overline{\left(\mathcal{T}_f^n \mathcal{T}_g^n\right) p\,(0) - \sum_{k<\ell} n^{-k} P_n\left(\mathcal{L}_k\left(f \text{⧗} g\right) p\right)(0)} \leqslant 2C\, n^{-\ell}$$

$$c_n = \frac{F_k(0)}{P_n\left(F_k\right)(0)} \leadsto 1 \Rightarrow \overline{c_n} \leqslant 3/2$$

$$\Rightarrow \overline{\sum_{k<\ell} n^{-k} P_n\left(F_k\right)(0)} = \overline{c_n \sum_{k<\ell} n^{-k} P_n\left(F_k\right)(0)} = \overline{c_n}\, \overline{\sum_{k<\ell} n^{-k} P_n\left(F_k\right)(0)} \leqslant 2C\, n^{-\ell}$$