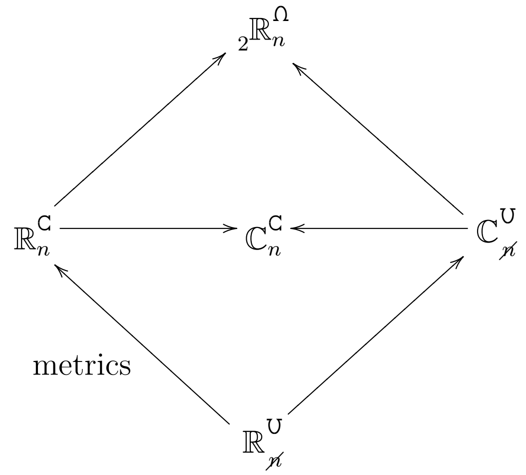
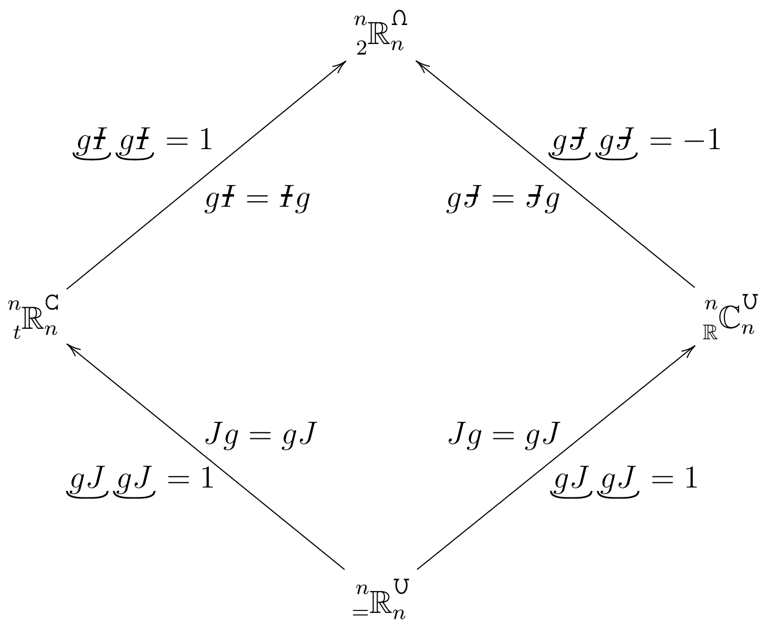
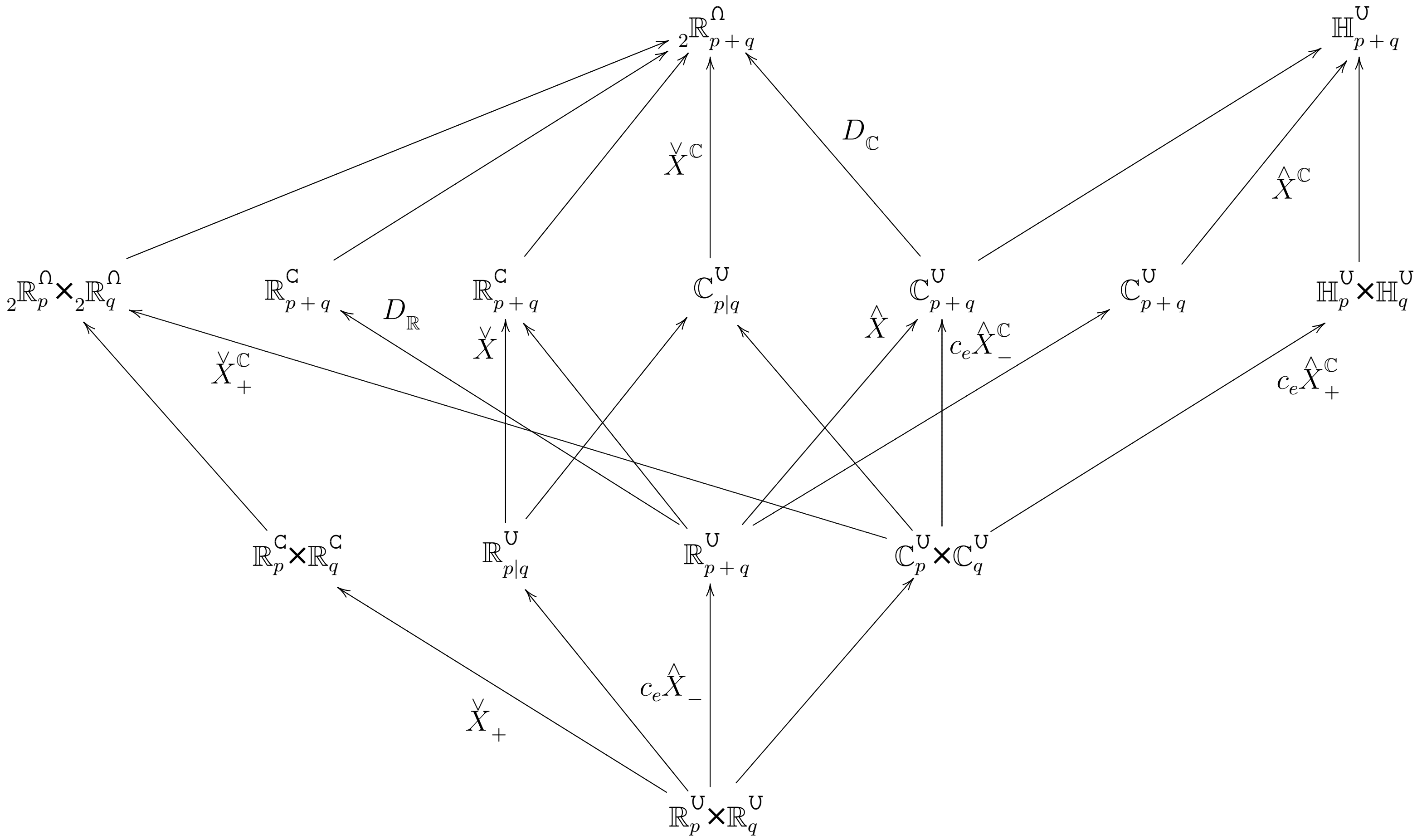
$$X = \mathbb{R}_n^{\mathrm{sym}}$$

$$X_+ = \mathbb{R}_p^{\mathrm{sym}} \times \mathbb{R}_q^{\mathrm{sym}}$$







$$ {}^{n}_{2}\mathbb{R}^{\cap}_{n} \ni \begin{array}{c|c} a & b \\ \hline c & -\overset{+}{a} \end{array} \quad \begin{cases} b = \overset{+}{b} \\ c = \overset{+}{c} \end{cases} $$

$$ {}^{n}_{\mathbb{R}}\mathbb{C}^{\cup}_{n} \ni \begin{array}{c|c} a & b \\ \hline -b & a \end{array} \quad \begin{cases} a = -\overset{+}{a} \\ b = \overset{+}{b} \end{cases} $$

$$ \begin{array}{c|c} a & b \\ \hline c & -\overset{+}{a} \end{array} = \mathcal{J} \begin{array}{c|c} a & b \\ \hline c & -\overset{+}{a} \end{array} \mathcal{J}^{-1} = \begin{array}{c|c} -\overset{+}{a} & -c \\ \hline -b & a \end{array} \Leftrightarrow \begin{cases} a = -\overset{+}{a} \\ c = -b \end{cases} $$

$$ {}^{n}_{t}\mathbb{R}^{\subset}_{n} \ni \begin{array}{c|c} a & 0 \\ \hline 0 & -\overset{+}{a} \end{array} $$

$$ \begin{array}{c|c} a & b \\ \hline c & -\overset{+}{a} \end{array} = \mathcal{I} \begin{array}{c|c} a & b \\ \hline c & -\overset{+}{a} \end{array} \mathcal{I} = \begin{array}{c|c} a & -b \\ \hline -c & -\overset{+}{a} \end{array} \Leftrightarrow \begin{cases} b = 0 \\ c = 0 \end{cases} $$

$$ {}^{n}_{=}\mathbb{R}^{\cup}_{n} \ni \begin{array}{c|c} a & 0 \\ \hline 0 & a \end{array} : \quad a = -\overset{+}{a} $$

$$\begin{array}{c|c} a & b \\ \hline -b & a \end{array} = J \begin{array}{c|c} a & b \\ \hline -b & a \end{array} J = \begin{array}{c|c} a & -b \\ \hline b & a \end{array} \Leftrightarrow b = 0$$

$$\begin{array}{c|c} a & 0 \\ \hline 0 & -\overset{+}{a} \end{array} = J \begin{array}{c|c} a & 0 \\ \hline 0 & -\overset{+}{a} \end{array} J = \begin{array}{c|c} -\overset{+}{a} & 0 \\ \hline 0 & a \end{array} \Leftrightarrow a = -\overset{+}{a}$$

$$\begin{cases} {}^{n}_{\mathbb{R}}\mathbb{C}^{\Cup}_{n} \\ {}^{n}_{t}\mathbb{R}^{\complement}_{n} \end{cases} = \frac{g \in {}^{n}_{2}\mathbb{R}^{\Omega}_{n}}{\begin{cases} g\text{Ɉ} = \text{Ɉ}g \\ g\text{Ɨ} = \text{Ɨ}g \end{cases}}$$

$$\begin{cases} \overset{*}{g}\text{Ɉ}g = \text{Ɉ} \\ g\text{Ɉ} = \text{Ɉ}g \end{cases} \Longrightarrow \overset{*}{g}g = I$$

$${}^{n}_{=}\mathbb{R}^{\Cup}_{n} = \frac{g \in \begin{cases} {}^{n}_{\mathbb{R}}\mathbb{C}^{\Cup}_{n} \\ {}^{n}_{t}\mathbb{R}^{\complement}_{n} \end{cases}}{Jg = gJ}$$

$$g \in {}^{n}_{2}\mathbb{R}^{\Omega}_{n} \Leftrightarrow z\,\text{Ɨ} \in {}^{n}_{2}\mathbb{R}^{\mho}_{n} \Leftrightarrow z = \begin{array}{c|c} a & b \\ \hline -\overset{*}{b} & d \end{array} \begin{cases} \overset{*}{a} = a \\ \overset{*}{d} = d \end{cases}$$

$$\overbrace{J+z}^{-1}\underbrace{J-z} = \overset{-1}{g} = \text{Ɉ}\,\overset{+}{g}\,\overset{-1}{\text{Ɉ}} = \text{Ɉ}\,\underbrace{J+\overset{+}{z}}\,\overbrace{J-\overset{+}{z}}^{-1}\,\overset{-1}{\text{Ɉ}} = \overbrace{\text{Ɨ}+\text{Ɉ}\overset{+}{z}}\,\overbrace{\text{Ɨ}-\text{Ɉ}\overset{+}{z}}^{-1}$$

$$J\text{Ɨ} - z\text{Ɨ} - J\text{Ɉ}\overset{+}{z} + z\text{Ɉ}\overset{+}{z} = \underbrace{J-z}\,\underbrace{\text{Ɨ}-\text{Ɉ}\overset{+}{z}} = \underbrace{J+z}\,\underbrace{\text{Ɨ}+\text{Ɉ}\overset{+}{z}} = J\text{Ɨ} + z\text{Ɨ} + J\text{Ɉ}\overset{+}{z} + z\text{Ɉ}\overset{+}{z}$$

$$z\text{Ɨ} = -J\text{Ɉ}\overset{+}{z} = \text{Ɨ}\overset{+}{z} = \overbrace{z\text{Ɨ}}^{+}$$

$$Jz = -zJ \Leftrightarrow z = \begin{array}{c|c} a & b \\ \hline -b & -a \end{array} \Leftrightarrow \begin{cases} a = \overset{*}{a} \\ b = \overset{*}{b} \end{cases}$$

$$\begin{array}{c|c} 0 & b \\ \hline -b & 0 \end{array} \in \begin{array}{c|c} 0 & {}^{n}\mathbb{R}_{n}^{\Cup} \\ \hline -\imath & 0 \end{array} \longrightarrow \frac{{}^{n}_{t}\mathbb{R}_{n}^{\subset}}{{}^{n}_{=}\mathbb{R}_{n}^{\cup}} \ni \begin{array}{c|c} \overbrace{1+b}^{-1}\overbrace{1-b} & 0 \\ \hline 0 & \underbrace{1-b}_{-1}\underbrace{1+b} \end{array}$$

$$\begin{array}{c|c} a & b \\ \hline \overset{+}{-b} & -d \end{array} \in {}^{n}_{2}\mathbb{R}_{n}^{\mathfrak{D}} Ɉ \longrightarrow {}^{n}_{2}\mathbb{R}_{n}^{\cap} \ni \overbrace{J-z}^{-1}\underbrace{J+z}$$

$$\begin{array}{c|c} a & 0 \\ \hline 0 & -a \end{array} \in \begin{array}{c|c} {}^{n}\mathbb{R}_{n}^{\Cup} & 0 \\ \hline 0 & \end{array} \longrightarrow \frac{{}^{n}_{\mathbb{R}}\mathbb{C}_{n}^{\cup}}{{}^{n}_{=}\mathbb{R}_{n}^{\cup}} \ni \begin{array}{c|c} \overbrace{1-a^2}\overbrace{1+a^2}^{-1} & -2a\overbrace{1+a^2}^{-1} \\ \hline 2a\underbrace{1+a^2}_{-1} & \underbrace{1-a^2}\underbrace{1+a^2}_{-1} \end{array}$$

$$z = \begin{array}{c|c} a & 0 \\ \hline 0 & -a \end{array} \Longrightarrow \overbrace{J-z}^{-1}\underbrace{J+z} = \begin{array}{c|c} \overbrace{1-a^2}\overbrace{1+a^2}^{-1} & -2a\overbrace{1+a^2}^{-1} \\ \hline 2a\underbrace{1+a^2}_{-1} & \underbrace{1-a^2}\underbrace{1+a^2}_{-1} \end{array} \in {}^{n}_{=}\mathbb{R}_{n}^{\cup} \neg {}^{n}_{\mathbb{R}}\mathbb{C}_{n}^{\cup}$$

$$g \in {}^{n}_{\mathbb{R}}\mathbb{C}_{n}^{\cup} \Leftrightarrow z = Ɉ\, z\, Ɉ = \begin{array}{c|c} a & -b \\ \hline b & -a \end{array} \Leftrightarrow b = 0$$

$$\text{LHS} = \begin{array}{c|c} -a & 1 \\ \hline 1 & a \end{array}^{-1} \begin{array}{c|c} a & 1 \\ \hline 1 & -a \end{array} = \begin{array}{c|c} -a\overbrace{1+a^2}^{-1} & \overbrace{1+a^2}^{-1} \\ \hline \overbrace{1+a^2}^{-1} & a\overbrace{1+a^2}^{-1} \end{array} \begin{array}{c|c} a & 1 \\ \hline 1 & -a \end{array} = \text{RHS}$$

$$z = \begin{array}{c|c} 0 & b \\ \hline -b & 0 \end{array} \Longrightarrow \overbrace{J-z}^{-1}\underbrace{J+z} = \begin{array}{c|c} \overbrace{1+b}^{-1}\overbrace{1-b} & 0 \\ \hline 0 & \underbrace{1-b}_{-1}\underbrace{1+b} \end{array} \in {}^{n}_{=}\mathbb{R}_{n}^{\cup} \neg {}^{n}_{t}\mathbb{R}_{n}^{\subset}$$

$$g \in {}^{n}_{t}\mathbb{R}_{n}^{\subset} \Leftrightarrow Ɨ\, z\, Ɨ = -z \Leftrightarrow z = \begin{array}{c|c} 0 & b \\ \hline -b & 0 \end{array} \Leftrightarrow a = 0$$

$$\text{LHS} = \begin{array}{c|c} 0 & 1-b \\ \hline 1+b & 0 \end{array}^{-1} \begin{array}{c|c} 0 & 1+b \\ \hline 1-b & 0 \end{array} = \begin{array}{c|c} 0 & \overbrace{1+b}^{-1} \\ \hline \underbrace{1-b}_{-1} & 0 \end{array}^{-1} \begin{array}{c|c} 0 & 1+b \\ \hline 1-b & 0 \end{array} = \text{RHS}$$

$$J = \begin{array}{c|c} 0 & 1 \\ \hline 1 & 0 \end{array} : \quad Ɉ = \begin{array}{c|c} 0 & 1 \\ \hline -1 & 0 \end{array} : \quad Ɨ = \begin{array}{c|c} 1 & 0 \\ \hline 0 & -1 \end{array}$$