$$\text{symbol } \mathcal{S} \xrightarrow[\text{stellig}]{|\cdot|} \mathbb{N}: \quad |s_n| = n$$

$$\text{Wort alg } W = < \mathcal{S}_0 : \mathcal{S}_1 : \cdots > = \bigcup_{\ell \geqslant 1} \mathcal{S} \times \cdot\cdot \times \mathcal{S} = \frac{s^1 \cdots s^\ell}{s^i \in \mathcal{S}} \ni w = s^1 \cdots s^\ell \text{ Wort der Lange } \ell \geqslant 1$$

$$s_n \in \mathcal{S}_n \Rightarrow \text{ n-Funktion } W \xleftarrow{s_n} W \times \cdot\cdot \times W: \quad s_n \, w_1 \cdot\cdot w_n = s_n \, w_1 \cdot\cdot w_n \text{ juxtaposition}$$

$$\text{0-symbol } s_0 \in O = \mathcal{S}_0 = \underline{W}$$

$$\text{1-symbol } s_1 \in \mathcal{S}_1 \Rightarrow W \xleftarrow{s_1} W: \quad w \curvearrowright s_1 w$$

$$\text{2-symbol } s_2 \in \mathcal{S}_2 \Rightarrow W \xleftarrow{s_2} W \times W: \quad z{:}w \curvearrowright s_2 zw$$

$$K\left(s^1 \cdots s^\ell\right) = |s^1| + \cdots + |s^\ell| - \ell \Rightarrow W \xrightarrow[\text{inv}]{K} \mathbb{Z} \Rightarrow K(zw) = K(z) + K(w)$$

$$\bar{\underline{W}} = \bar{O} = \bar{\mathcal{S}}_0 \text{ Terme=ableitbare Worte}$$

$$2' \, 1' \, 210 \, 0' \, 0^* \text{ term}: \quad 0|10|0'|210\,0'|\,1'\,210\,0'\,|\,0^*\,|\,2'\,1'\,210\,0'\,0^*$$

$$t \in \bar{O} \text{ ableitbar } \Rightarrow K(t) = -1$$

$$O \subset V = \frac{t \in W}{K(t) = -1} \underset{\text{abg}}{\subset} W$$

$$K(o) = 0 - 1 = -1: \quad s_n \in \mathcal{S}_n: \quad t_1 \cdots t_n \in V$$

$$\Rightarrow K\left(s_n t_1 \cdots t_n\right) = K\left(s_n t_1 \cdot\cdot t_n\right) = \underbrace{K(s_n)}_{n-1} + \underbrace{K(t_1)}_{-1} + \cdots + \underbrace{K(t_n)}_{-1} = -1 \Rightarrow s_n \, t_1 \cdots t_n \in V$$

$$\overset{\text{Ind}}{\underset{\text{Satz}}{\Longrightarrow}} \bar{O} \subset V$$

$$\bar{O} \ni t = zw \underset{*}{\Rightarrow} \bigvee_{t_k}^{\bar{O}} w = t_1 \cdots t_m : \quad \text{Endstuck von Term ist Term-Produkt}$$

$$V = \frac{t \in \bar{O}}{\bigwedge_{t = zw} w \text{ term product}} \underset{\text{abg}}{\subset} W$$

$$O \ni o \text{ kein product} : \quad s_n \in \mathcal{S}_n : \quad t_1 \cdots t_n \in V : \quad s_n\, t_1 \cdots t_n = zw \underset{\text{Beh}}{\Rightarrow} w \text{ term product}$$

$$\text{If} \bigvee_{1 \leqslant i \leqslant n} \begin{cases} z = s_n t_1 \cdot\cdot t_{i-} \\ w = t_i \cdot\cdot t_n \text{ term product} \end{cases} \quad : \quad \text{If} \bigvee_{1 \leqslant i \leqslant n} \bigvee_{t_i = z_i w_i}^{\text{prop fact}} \begin{cases} z = s_n t_1 \cdot\cdot t_{i-} z^i \\ w = w^i t_{i+} \cdot\cdot t_n \end{cases}$$

$$t_i \in V \Rightarrow w_i = t_i^1 \cdot\cdot t_i^m \text{ term product } \Rightarrow w = t_i^1 \cdot\cdot t_i^m\, t_{i+} \cdot\cdot t_n \text{ term product}$$

$$\overset{\text{Ind}}{\underset{\text{Satz}}{\Rightarrow}} \bar{O} \subset V$$

$$zw \in \bar{O} \Rightarrow z \notin \bar{O} \text{ echtes Anfangsstuck von Term ist kein Term}$$

$$\text{LEM} \Rightarrow \bigvee_{t_k}^{\text{terms}} w = t_1 \cdots t_m \Rightarrow z\, t_1 \cdots t_m \in \bar{O}$$

$$\Rightarrow -1 = K\left(zt_1 \cdots t_m\right) = K(z) + \underbrace{K\left(t_1\right)}_{-1} + \cdots + \underbrace{K(t_m)}_{-1} = K(z) - m \Rightarrow K(z) = m - 1 \geqslant 0 \Rightarrow z \notin \bar{O}$$

$\bar{O}$ Peano

$$\text{Eind}\ \ s_n\, t^1 \cdots t^n = \tilde{s}_m\, \tilde{t}^1 \cdots \tilde{t}^m : \quad t^i \in \bar{O} \ni \tilde{t}^j \Rightarrow \underbrace{s_n = \tilde{s}_m}_{\text{letters}} : \quad n = m : \quad t^1 \cdots t^n = \tilde{t}^1 \cdots \tilde{t}^n$$

$$\lightning\ t^1 \neq \tilde{t}^1 \underset{\text{OE}}{\Rightarrow} \tilde{t}^1 = t^1 b \Rightarrow t^1 \text{ echtes Anfangsstuck von } \tilde{t}^1\ \lightning \Rightarrow t^1 = \tilde{t}^1$$

$$\Rightarrow t^2 \cdots t^n = \tilde{t}^2 \cdots \tilde{t}^n \underset{\text{Ind}}{\Rightarrow} t^i = \tilde{t}^i$$

$t \in \bar{O}$ ableitbar in $\bar{O}$

$$\bigvee_{\text{Abl}} t_1 | \cdot\cdot | t_n | t \text{ mit } t_i \in W$$

$$\bigwedge_{1 \leqslant m \leqslant n} t_1 | \cdot\cdot | t_m \text{ Ableitung } \Rightarrow t_m \in \bar{O}$$