

$$\begin{aligned} \mathbb{K}\Delta^0 \ni \mathfrak{b} &= \mathfrak{b}_+ \sqcup \mathfrak{b}_- \ni \mathfrak{b} = \mathfrak{b}_+ + \mathfrak{b}_- \\ (-1)^{ki} \mathfrak{b} * \mathfrak{b} * \mathfrak{b} &+ (-1)^{ij} \mathfrak{b} * \mathfrak{b} * \mathfrak{b} + (-1)^{jk} \mathfrak{b} * \mathfrak{b} * \mathfrak{b} = 0 \\ \mathfrak{b} * \mathfrak{b} &+ (-1)^{ij} \mathfrak{b} * \mathfrak{b} = 0 \end{aligned}$$

Grassmann envelope

$$\Lambda_+ \Delta^0 \ni \Lambda \mathfrak{b} = \Lambda_+ \mathfrak{z} \mathfrak{b}_+ \sqcup \Lambda_- \mathfrak{z} \mathfrak{b}_- \ni \Xi \mathfrak{z} \mathfrak{b}_+ + \eta \mathfrak{z} \mathfrak{b}_- \text{ super Lie algebra}$$

$$\lambda \overline{\Xi \mathfrak{z} \mathfrak{b}_+ + \eta \mathfrak{z} \mathfrak{b}_-} = \overline{\lambda \xi \mathfrak{z} \mathfrak{b}_+} + \overline{\lambda \eta \mathfrak{z} \mathfrak{b}_-}$$

$$\overline{\alpha \mathfrak{z} \mathfrak{b}} * \overline{\gamma \mathfrak{z} \mathfrak{b}_\ell} = (-1)^{jk} \overline{\alpha \gamma \mathfrak{z} \mathfrak{b} * \mathfrak{b}_\ell}$$

$$\Lambda \mathfrak{b} = \Lambda_+ \mathfrak{z} \mathfrak{b}_+ \sqcup \Lambda_- \mathfrak{z} \mathfrak{b}_- \text{ Lie algebra}$$

$$\begin{aligned} \text{asymm : } \underbrace{\alpha \mathfrak{z} \mathfrak{b}}_i * \underbrace{\beta \mathfrak{z} \mathfrak{b}}_j + \underbrace{\beta \mathfrak{z} \mathfrak{b}}_j * \underbrace{\alpha \mathfrak{z} \mathfrak{b}}_i &= (-1)^{ij} \overline{\alpha \beta \mathfrak{z} \mathfrak{b} * \mathfrak{b}} + (-1)^{ji} \overline{\beta \alpha \mathfrak{z} \mathfrak{b} * \mathfrak{b}} \\ &= (-1)^{ij} \overline{\alpha \beta \mathfrak{z} \mathfrak{b} * \mathfrak{b}} + \underbrace{(-1)^{ji} \overline{\mathfrak{b} * \mathfrak{b}}}_{=0} \end{aligned}$$

$$\begin{aligned} \text{atrans : } &(-1)^{ij} (-1)^{jk} (-1)^{ki} \left(\overline{\alpha \mathfrak{z} \mathfrak{b}} * \overline{\beta \mathfrak{z} \mathfrak{b}} * \overline{\gamma \mathfrak{z} \mathfrak{b}} + \overline{\beta \mathfrak{z} \mathfrak{b}} * \overline{\gamma \mathfrak{z} \mathfrak{b}} * \overline{\alpha \mathfrak{z} \mathfrak{b}} + \overline{\gamma \mathfrak{z} \mathfrak{b}} * \overline{\alpha \mathfrak{z} \mathfrak{b}} * \overline{\beta \mathfrak{z} \mathfrak{b}} \right) \\ &= (-1)^{ij} (-1)^{ki} \overline{\alpha \mathfrak{z} \mathfrak{b}} * \overline{\beta \gamma \mathfrak{z} \mathfrak{b} * \mathfrak{b}} + (-1)^{ij} (-1)^{jk} \overline{\beta \mathfrak{z} \mathfrak{b}} * \overline{\gamma \alpha \mathfrak{z} \mathfrak{b} * \mathfrak{b}} + (-1)^{jk} (-1)^{ki} \overline{\gamma \mathfrak{z} \mathfrak{b}} * \overline{\alpha \beta \mathfrak{z} \mathfrak{b} * \mathfrak{b}} \\ &= \overline{\alpha \beta \gamma \mathfrak{z} \mathfrak{b} * \mathfrak{b} * \mathfrak{b}} + \overline{\beta \gamma \alpha \mathfrak{z} \mathfrak{b} * \mathfrak{b} * \mathfrak{b}} + \overline{\gamma \alpha \beta \mathfrak{z} \mathfrak{b} * \mathfrak{b} * \mathfrak{b}} \\ &= \overline{\alpha \beta \gamma \mathfrak{z} \mathfrak{b} * \mathfrak{b} * \mathfrak{b}} + (-1)^{ki} (-1)^{ji} \overline{\mathfrak{b} * \mathfrak{b} * \mathfrak{b}} + (-1)^{ki} (-1)^{kj} \overline{\mathfrak{b} * \mathfrak{b} * \mathfrak{b}} \\ &= \overline{(-1)^{ki} \alpha \beta \gamma \mathfrak{z} \mathfrak{b} * \mathfrak{b} * \mathfrak{b}} + \overline{(-1)^{ji} \mathfrak{b} * \mathfrak{b} * \mathfrak{b}} + \overline{(-1)^{kj} \mathfrak{b} * \mathfrak{b} * \mathfrak{b}} = 0 \end{aligned}$$