

real

Bert I:2 $Z_{\mathbb{C}} \supset B_{\mathbb{C}}$

$$Z_{\mathbb{R}} = i^n \mathbb{R}_n^{\mathfrak{D}} = {}^n \mathbb{R}_n^{\mathfrak{U}} \supset B_{\mathbb{R}}$$

$$G_{\mathbb{C}} = {}^n \mathbb{H}_n^{\mathfrak{D}} = SO_{2n}^*$$

$$G_{\mathbb{R}} = {}^n \mathbb{C}_n^{\mathfrak{D}} = SO_{n:n}$$

$$G_{\mathbb{C}}^c = {}^{n:n} \mathbb{R}_{n:n}^{\mathfrak{D}} = SO_{n:n}$$

Bert II:0 Cau $Z_{\mathbb{C}} \supset B_{\mathbb{C}}$

$$Z_{\mathbb{R}} = {}^n \mathbb{R}_n^{\mathfrak{U}} \supset B_{\mathbb{R}}$$

$$G_{\mathbb{C}} = G_{\mathbb{C}}^c = {}^{2n} \mathbb{R}_{2n}^{\Omega} = Sp_n(\mathbb{R})$$

$$G_{\mathbb{R}} = {}^n \mathbb{R}_n^{\mathfrak{C}} = {}^n \mathbb{R}_n^{\mathfrak{C}} \times \mathbb{R}^{\times} = GL_n(\mathbb{R})$$

$$K_{\mathbb{C}} = {}^n \mathbb{C}_n^{\mathfrak{U}} = U_n$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}} = {}^n \mathbb{R}_n^{\mathfrak{U}} = O_n$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}}$$

$$\text{Bert II:1 } Z_{\mathbb{C}} = {}^p C_q \supset B_{\mathbb{C}}$$

$$Z_{\mathbb{R}} = {}^p \mathbb{R}_q \supset B_{\mathbb{R}}$$

$$G_{\mathbb{C}} = {}^{p:q} C_{p:q}^{\cup} = U_{p:q}$$

$$G_{\mathbb{R}} = {}^{p:q} \mathbb{R}_{p:q}^{\cup} = U_{p:q}(\mathbb{R}) = O_{p:q}$$

$$G_{\mathbb{C}}^c = {}^{p+q} C_{p+q}^{\cup} = SL_{p+q}(\mathbb{R})$$

$$K_{\mathbb{C}} = {}^p C_p^{\cup} \times {}^q C_q^{\cup} = U_p \times U_q$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}} = {}^p \mathbb{R}_p^{\cup} \times {}^q \mathbb{R}_q^{\cup} = O_p \times O_q$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}}$$

$$\text{Bert IV:0 } Z_{\mathbb{C}} \supset B_{\mathbb{C}}$$

$$Z_{\mathbb{R}} = \mathbb{R} \times \mathbb{R}^{n-1} = \mathbb{R}^{1:n-1} \supset B_{\mathbb{R}}$$

$$G_{\mathbb{C}} = G_{\mathbb{C}}^c = {}_{2:n} \mathbb{R}^{2:n} = SO_{2:n}$$

$$G_{\mathbb{R}} = {}_{1:n-1} \mathbb{R}^{1:n-1} \times \mathbb{R}^{\times} = SO_{1:n-1} \times \mathbb{R}^{\times}$$

$$K_{\mathbb{C}}$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}}$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}}$$

$$\text{Bert IV:1}$$

$$G_{\mathbb{C}} = {}_n \mathbb{R}^n \times \mathbb{T} = SO_n \times \mathbb{T}$$

$$G_{\mathbb{R}} = {}_{n-1} \mathbb{R}^{n-1} = SO_{n-1}$$

Bert IV:2 $Z_{\mathbb{C}} \supset B_{\mathbb{C}}$

$$Z_{\mathbb{R}} = \mathbb{R}^{n-p} \supset B_{\mathbb{R}}$$

$$G_{\mathbb{C}} = \mathfrak{O}_p \mathbb{R}^p \times_{2:n-p} \mathfrak{O}_{2:n-p} \mathbb{R}^{2:n-p} = SO_p \times SO_{2:n-p}$$

$$G_{\mathbb{R}} = \mathfrak{O}_{p-1} \mathbb{R}^{p-1} \times_{1:n-p} \mathfrak{O}_{1:n-p} \mathbb{R}^{1:n-p} = SO_{p-1} \times SO_{1:n-p}$$

$$K_{\mathbb{C}}$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}}$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}}$$

Bert IV:3 $Z_{\mathbb{C}} \supset B_{\mathbb{C}}$

$$Z_{\mathbb{R}} = \mathbb{R}^{n-1} \supset B_{\mathbb{R}}$$

$$G_{\mathbb{C}} = \mathfrak{O}_{2:n-1} \mathbb{R}^{2:n-1} = SO_{2:n-1}$$

$$G_{\mathbb{R}} = \mathfrak{O}_{1:n-1} \mathbb{R}^{1:n-1} = SO_{1:n-1}$$

$$K_{\mathbb{C}}$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}}$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}}$$

Bert IV:5

$$G_{\mathbb{C}} = {}_{2:p+q}\mathbb{R}^{2:p+q} = SO_{2:p+q}$$
$$G_{\mathbb{R}} = {}_{1:p}\mathbb{R}^{1:p} \times {}_{1:q}\mathbb{R}^{1:q} = SO_{1:p} \times SO_{1:q}$$
$$G_{\mathbb{C}}^c = {}_{p+1:q+1}\mathbb{R}^{p+1:q+1} = SO_{p+1:q+1}$$

$K_{\mathbb{C}}$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}}$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}}$$

complex

Bert I:0 Cau

$$Z_{\mathbb{C}} = {}^n\mathbb{C}_n \supset B_{\mathbb{C}}$$

$$Z_{\mathbb{R}} = {}^n\mathbb{C}_n^{\cup} \supset B_{\mathbb{R}}$$

$$G_{\mathbb{C}} = {}^{n:n}\mathbb{C}_{n:n}^{\cup} = U_{n:n}$$

$$G_{\mathbb{R}} = {}^n\mathbb{C}_n^{\mathbb{C}} \times \mathbb{R}^{\times} = SL_n(\mathbb{C}) \times \mathbb{R}^{\times}$$

$$G_{\mathbb{R}} : \times G_{\mathbb{C}} G_{\mathbb{C}}^c = G_{\mathbb{C}}$$

$$K_{\mathbb{C}} = {}^n\mathbb{C}_n^{\cup} \times {}^n\mathbb{C}_n^{\cup} = U_n \times U_n$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}} = {}^n\mathbb{C}_n^{\cup} = U_n \text{ diag}$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}} = {}^n\mathbb{C}_n^{\cup} = U_n \text{ off-diag} = S_{\mathbb{R}}$$

Bert I:1 prod

$$Z_{\mathbb{C}} = {}^p\mathbb{C}_q \times {}^p\mathbb{C}_q \supset B_{\mathbb{C}} = B_{\mathbb{R}} \times B_{\mathbb{R}}$$

$$Z_{\mathbb{R}} = {}^p\mathbb{C}_q \text{ real} \supset B_{\mathbb{R}}$$

$$G_{\mathbb{C}} = {}^{p,q}\mathbb{C}_{p,q}^{\mathbb{U}} \times {}^{p,q}\mathbb{C}_{p,q}^{\mathbb{U}} = U_{p,q} \times U_{p,q}$$

$$G_{\mathbb{R}} = {}^{p,q}\mathbb{C}_{p,q}^{\mathbb{U}} \text{ diag} = U_{p,q}$$

$$G_{\mathbb{R}} : \times G_{\mathbb{C}} = {}^{p,q}\mathbb{C}_{p,q}^{\mathbb{U}} \text{ space} = U_{p,q}$$

$$G_{\mathbb{C}}^c = G_{\mathbb{R}}^c = {}^{p+q}\mathbb{C}_{p+q} = SL_{p+q}(\mathbb{C})$$

$$K_{\mathbb{C}} = {}^p\mathbb{C}_p^{\mathbb{U}} \times {}^q\mathbb{C}_q^{\mathbb{U}} \times {}^p\mathbb{C}_p^{\mathbb{U}} \times {}^q\mathbb{C}_q^{\mathbb{U}} = U_p(\mathbb{C}) \times U_q(\mathbb{C}) \times U_p \times U_q$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}} = {}^p\mathbb{C}_p^{\mathbb{U}} \times {}^q\mathbb{C}_q^{\mathbb{U}} = U_p \times U_q \text{ diag}$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}} = {}^p\mathbb{C}_p^{\mathbb{U}} \times {}^q\mathbb{C}_q^{\mathbb{U}} = U_p \times U_q = S_{\mathbb{R}} \times S_{\mathbb{R}}$$

Bert II:2 prod good

$$Z_{\mathbb{C}} = {}^n\mathbb{C}_n^{\mathfrak{B}} \times {}^n\mathbb{C}_n^{\mathfrak{B}} \supset B_{\mathbb{C}}$$

$$Z_{\mathbb{R}} = {}^n\mathbb{C}_n^{\mathfrak{B}} \text{ real} \supset B_{\mathbb{R}}$$

$$G_{\mathbb{C}} = {}^{2n}\mathbb{R}_{2n}^{\Omega} \times {}^{2n}\mathbb{R}_{2n}^{\Omega} = Sp_n(\mathbb{R}) \times Sp_n(\mathbb{R})$$

$$G_{\mathbb{R}} = {}^{2n}\mathbb{R}_{2n}^{\Omega} \text{ diag} = Sp_n(\mathbb{R})$$

$$G_{\mathbb{C}}^c = {}^{2n}\mathbb{C}_{2n}^{\Omega} = Sp_n(\mathbb{C})$$

$$K_{\mathbb{C}} = {}^n\mathbb{C}_n^{\mathbb{U}} \times {}^n\mathbb{C}_n^{\mathbb{U}} = U_n \times U_n$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}} = {}^n\mathbb{C}_n^{\mathbb{U}} = U_n \text{ diag}$$

Bert III:2 prod

$$Z_{\mathbb{C}} = {}^n\mathbb{C}_n^{\mathfrak{D}} \times {}^n\mathbb{C}_n^{\mathfrak{D}} \supset B_{\mathbb{C}}$$

$$Z_{\mathbb{R}} = {}^n\mathbb{C}_n^{\mathfrak{D}} \text{ real} \supset B_{\mathbb{R}}$$

$$G_{\mathbb{C}} = {}^n\mathbb{H}_n^{\mathfrak{D}} \times {}^n\mathbb{H}_n^{\mathfrak{D}} = SO_{2n}^* \times SO_{2n}^*$$

$$G_{\mathbb{R}} = {}^n\mathbb{H}_n^{\mathfrak{D}} = SO_{2n}^* \text{ diag}$$

$$G_{\mathbb{C}}^c = {}^{2n}\mathbb{C}_{2n}^{\mathfrak{D}} = SO_{2n}(\mathbb{C})$$

$$K_{\mathbb{C}} = {}^n\mathbb{C}_n^{\mathfrak{U}} \times {}^n\mathbb{C}_n^{\mathfrak{U}} = U_n \times U_n$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}} = {}^n\mathbb{C}_n^{\mathfrak{U}} = U_n \text{ diag}$$

Bert IV: prod

$$Z_{\mathbb{C}} = \mathbb{C}^n \times \mathbb{C}^n \supset B_{\mathbb{C}}$$

$$Z_{\mathbb{R}} = \mathbb{C}^n \text{ real} \supset B_{\mathbb{R}}$$

$$G_{\mathbb{C}} = {}_{2,n}\mathbb{R}^{2,n} \times {}_{2,n}\mathbb{R}^{2,n} = SO_{2,n} \times SO_{2,n}$$

$$G_{\mathbb{R}} = {}_{2,n}\mathbb{R}^{2,n} = SO_{2,n}$$

$$G_{\mathbb{C}}^c = {}_{2+n}\mathbb{C}^{2+n} = SO_{2+n}(\mathbb{C})$$

$$K_{\mathbb{C}}$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}}$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}}$$

quaternion

Bert I:3 good

$$Z_{\mathbb{C}} \supset B_{\mathbb{C}}$$

$$Z_{\mathbb{R}} = {}^n\mathbb{H}_n^{\mathbb{U}} \supset B_{\mathbb{R}}$$

$$G_{\mathbb{C}} = {}^{4n}\mathbb{R}_{4n}^{\Omega} = Sp_{4n}(\mathbb{R})$$

$$G_{\mathbb{R}} = {}^{2n}\mathbb{C}_{2n}^{\Omega} = Sp_n(\mathbb{C})$$

$$G_{\mathbb{C}}^c = {}^{4n}\mathbb{C}_{4n}^{\Omega} \cap {}^{2n:2n}\mathbb{C}_{2n:2n}^{\mathbb{U}} = Sp_{n:n}$$

$$K_{\mathbb{C}} = {}^{2n}\mathbb{C}_{2n}^{\mathbb{U}} = U_{2n}$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}} = {}^n\mathbb{H}_n^{\mathbb{U}} = Sp_n$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}}$$

Bert III:0 Cau

$$Z_{\mathbb{C}} \supset B_{\mathbb{C}}$$

$$Z_{\mathbb{R}} = {}^n\mathbb{H}_n^{\mathbb{U}} \supset B_{\mathbb{R}}$$

$$G_{\mathbb{C}} = G_{\mathbb{C}}^c = {}^{2n}\mathbb{H}_{2n}^{\mathbb{D}} = SO_{2n}^*$$

$$G_{\mathbb{R}} = {}^n\mathbb{H}_n^{\mathbb{C}} \times \mathbb{R}^{\times} = SL_n(\mathbb{H}) \times \mathbb{R}^{\times} = SU_{2n}^* \times \mathbb{R}^{\times}$$

$$K_{\mathbb{C}} = {}^{2n}\mathbb{C}_{2n}^{\mathbb{U}} = U_{2n}$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}} = {}^{2n}\mathbb{C}_{2n}^{\Omega * U} = {}^{2n}\mathbb{C}_{2n}^{\Omega} \cap {}^{2n}\mathbb{C}_{2n}^{\mathbb{U}} = Sp_n$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}}$$

Bert III:1

$$Z_{\mathbb{C}} \supset B_{\mathbb{C}}$$

$$Z_{\mathbb{R}} = {}^p\mathbb{H}_q \supset B_{\mathbb{R}}$$

$$G_{\mathbb{C}} = {}^{2p:2q}\mathbb{C}_{2p:2q}^{\mathbb{U}} = U_{2p:2q}$$

$$G_{\mathbb{R}} = {}^{2p:2q}\mathbb{C}_{2p:2q}^{\mathbb{U}} \cap {}^{2(p+q)}\mathbb{C}_{2(p+q)}^{\mathbb{N}} = Sp_{p:q}$$

$$G_{\mathbb{C}}^c = {}^{p+q}\mathbb{C}_{p+q}^{\mathbb{H}} = SU_{2(p+q)}^*$$

$$K_{\mathbb{C}} = {}^{2p}\mathbb{C}_{2p}^{\mathbb{U}} \times {}^{2q}\mathbb{C}_{2q}^{\mathbb{U}} = U_{2p} \times U_{2q}$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}} = {}^p\mathbb{H}_p^{\mathbb{U}} \times {}^q\mathbb{H}_q^{\mathbb{U}} = Sp_p \times Sp_q$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}}$$

octonion

Bert V:1 cpt

$$G_{\mathbb{C}} = E_{6:-14} \times \mathbb{T}$$

$$G_{\mathbb{R}} = F_{4:-20}$$

$$G_{\mathbb{C}}^c = E_{6:-26}$$

$$K_{\mathbb{C}}$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}}$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}}$$

$$S = \text{cpt}$$

Bert V:2

$$\begin{aligned}G_{\mathbb{C}} &= {}_{6:2}\mathbb{U}\mathbb{C}^{6:2} = SU_{6:2} \\G_{\mathbb{R}} &= {}_{6:2}\mathbb{U}\mathbb{C}^{6:2} \cap {}_8\mathbb{N}\mathbb{C}^8 = Sp_{3:1} \\&G_{\mathbb{C}}^c \\&K_{\mathbb{C}} \\K_{\mathbb{R}} &= K_{\mathbb{C}} \cap G_{\mathbb{R}} \\&K_{\mathbb{R}} : \times K_{\mathbb{C}}\end{aligned}$$

Bert V: $\text{prod } Z_{\mathbb{C}} = \mathbb{O}_{\mathbb{C}}^{1 \times 2} \times \mathbb{O}_{\mathbb{C}}^{1 \times 2}$

$$Z_{\mathbb{R}} = \mathbb{O}_{\mathbb{C}}^{1 \times 2} \text{ real}$$

$$\begin{aligned}G_{\mathbb{C}} &= E_{6:-14} \times E_{6:-14} \\G_{\mathbb{R}} &= E_{6:-14} \text{ diag} \\&G_{\mathbb{C}}^c = E_6 \\&K_{\mathbb{C}} \\K_{\mathbb{R}} &= K_{\mathbb{C}} \cap G_{\mathbb{R}} \\&K_{\mathbb{R}} : \times K_{\mathbb{C}}\end{aligned}$$

Bert VI:0 Cau $Z_{\mathbb{C}}$

$$Z_{\mathbb{R}} = {}^3\mathbb{O}_3^{\vartheta}$$

$$G_{\mathbb{C}} = E_{7:-25} = G_{\mathbb{C}}^c$$

$$G_{\mathbb{R}} = E_{6:-26} \times \mathbb{R}^{\times}$$

$$K_{\mathbb{C}}$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}}$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}}$$

Bert VI: prod $Z_{\mathbb{C}} = {}^3\mathbb{O}_3^{\vartheta} \times {}^3\mathbb{O}_3^{\vartheta}$

$$Z_{\mathbb{R}} = {}^3\mathbb{O}_3^{\vartheta} \text{ real}$$

$$G_{\mathbb{C}} = E_{7:-25} \times E_{7:-25}$$

$$G_{\mathbb{R}} = E_{7:-25} \text{ diag}$$

$$G_{\mathbb{C}}^c = E_7$$

$$K_{\mathbb{C}}$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}}$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}}$$

Bert VI:3

$$G_{\mathbb{C}} = E_{7:-25}$$

$$G_{\mathbb{R}} = {}^4\mathbb{H}_4^{\mathbb{C}} = SU_8^*$$

$$G_{\mathbb{C}}^c = E_{7:7}$$

total

I0 Cau

$$Z_{\mathbb{C}} = {}^n\mathbb{C}_n \supset B_{\mathbb{C}}$$

$$Z_{\mathbb{R}} = {}^n\mathbb{C}_n^{\mathfrak{U}} \supset B_{\mathbb{R}}$$

$$G_{\mathbb{C}} = {}^{n:n}\mathbb{C}_{n:n}^{\mathfrak{U}} = U_{n:n}$$

$$G_{\mathbb{R}} = {}^n\mathbb{C}_n^{\mathfrak{C}} \times \mathbb{R}^{\times} = SL_n(\mathbb{C}) \times \mathbb{R}^{\times}$$

$$G_{\mathbb{R}} : \times G_{\mathbb{C}} G_{\mathbb{C}}^c = G_{\mathbb{C}}$$

$$K_{\mathbb{C}} = {}^n\mathbb{C}_n^{\mathfrak{U}} \times {}^n\mathbb{C}_n^{\mathfrak{U}} = U_n \times U_n$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}} = {}^n\mathbb{C}_n^{\mathfrak{U}} = U_n \text{ diag}$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}} = {}^n\mathbb{C}_n^{\mathfrak{U}} = U_n \text{ off-diag} = S_{\mathbb{R}}$$

I1 prod

$$Z_{\mathbb{C}} = {}^p\mathbb{C}_q \times {}^p\mathbb{C}_q \supset B_{\mathbb{C}} = B_{\mathbb{R}} \times B_{\mathbb{R}}$$

$$Z_{\mathbb{R}} = {}^p\mathbb{C}_q \text{ real} \supset B_{\mathbb{R}}$$

$$G_{\mathbb{C}} = {}^{p:q}\mathbb{C}_{p:q}^{\mathfrak{U}} \times {}^{p:q}\mathbb{C}_{p:q}^{\mathfrak{U}} = U_{p:q} \times U_{p:q}$$

$$G_{\mathbb{R}} = {}^{p:q}\mathbb{C}_{p:q}^{\mathfrak{U}} \text{ diag} = U_{p:q}$$

$$G_{\mathbb{R}} : \times G_{\mathbb{C}} = {}^{p:q}\mathbb{C}_{p:q}^{\mathfrak{U}} \text{ space} = U_{p:q}$$

$$G_{\mathbb{C}}^c = G_{\mathbb{R}}^c = {}^{p+q}\mathbb{C}_{p+q}^{\mathfrak{C}} = SL_{p+q}(\mathbb{C})$$

$$K_{\mathbb{C}} = {}^p\mathbb{C}_p^{\mathfrak{U}} \times {}^q\mathbb{C}_q^{\mathfrak{U}} \times {}^p\mathbb{C}_p^{\mathfrak{U}} \times {}^q\mathbb{C}_q^{\mathfrak{U}} = U_p(\mathbb{C}) \times U_q(\mathbb{C}) \times U_p \times U_q$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}} = {}^p\mathbb{C}_p^{\mathfrak{U}} \times {}^q\mathbb{C}_q^{\mathfrak{U}} = U_p \times U_q \text{ diag}$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}} = {}^p\mathbb{C}_p^{\mathfrak{U}} \times {}^q\mathbb{C}_q^{\mathfrak{U}} = U_p \times U_q = S_{\mathbb{R}} \times S_{\mathbb{R}}$$

I2

$$Z_{\mathbb{C}} \supset B_{\mathbb{C}}$$

$$Z_{\mathbb{R}} = i {}^n\mathbb{R}_n^{\mathfrak{D}} = {}^n\mathbb{R}_n^{\mathfrak{U}} \supset B_{\mathbb{R}}$$

$$G_{\mathbb{C}} = {}^n\mathbb{H}_n^{\mathfrak{D}} = SO_{2n}^*$$

$$G_{\mathbb{R}} = {}^n\mathbb{C}_n^{\mathfrak{D}} = SO_{n:n}$$

$$G_{\mathbb{C}}^c = {}^{n:n}\mathbb{R}_{n:n}^{\mathfrak{D}} = SO_{n:n}$$

I3 good

$$Z_{\mathbb{C}} \supset B_{\mathbb{C}}$$

$$Z_{\mathbb{R}} = {}^n\mathbb{H}_n^{\mathbb{U}} \supset B_{\mathbb{R}}$$

$$G_{\mathbb{C}} = {}^{4n}\mathbb{R}_{4n}^{\Omega} = Sp_{4n}(\mathbb{R})$$

$$G_{\mathbb{R}} = {}^{2n}\mathbb{C}_{2n}^{\Omega} = Sp_n(\mathbb{C})$$

$$G_{\mathbb{C}}^c = {}^{4n}\mathbb{C}_{4n}^{\Omega} \cap {}^{2n:2n}\mathbb{C}_{2n:2n}^{\mathbb{U}} = Sp_{n:n}$$

$$K_{\mathbb{C}} = {}^{2n}\mathbb{C}_{2n}^{\mathbb{U}} = U_{2n}$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}} = {}^n\mathbb{H}_n^{\mathbb{U}} = Sp_n$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}}$$

II0 Cau

$$Z_{\mathbb{C}} \supset B_{\mathbb{C}}$$

$$Z_{\mathbb{R}} = {}^n\mathbb{R}_n^{\mathbb{U}} \supset B_{\mathbb{R}}$$

$$G_{\mathbb{C}} = G_{\mathbb{C}}^c = {}^{2n}\mathbb{R}_{2n}^{\Omega} = Sp_n(\mathbb{R})$$

$$G_{\mathbb{R}} = {}^n\mathbb{R}_n^{\mathbb{C}} = {}^n\mathbb{R}_n^{\mathbb{C}} \times \mathbb{R}^{\times} = GL_n(\mathbb{R})$$

$$K_{\mathbb{C}} = {}^n\mathbb{C}_n^{\mathbb{U}} = U_n$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}} = {}^n\mathbb{R}_n^{\mathbb{U}} = O_n$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}}$$

III

$$Z_{\mathbb{C}} = {}^p\mathbb{C}_q \supset B_{\mathbb{C}}$$

$$Z_{\mathbb{R}} = {}^p\mathbb{R}_q \supset B_{\mathbb{R}}$$

$$G_{\mathbb{C}} = {}^{p:q}\mathbb{C}_{p:q}^{\mathbb{U}} = U_{p:q}$$

$$G_{\mathbb{R}} = {}^{p:q}\mathbb{R}_{p:q}^{\mathbb{U}} = U_{p:q}(\mathbb{R}) = O_{p:q}$$

$$G_{\mathbb{C}}^c = {}^{p+q}\mathbb{C}_{p+q}^{\mathbb{R}} = SL_{p+q}(\mathbb{R})$$

$$K_{\mathbb{C}} = {}^p\mathbb{C}_p^{\mathbb{U}} \times {}^q\mathbb{C}_q^{\mathbb{U}} = U_p \times U_q$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}} = {}^p\mathbb{R}_p^{\mathbb{U}} \times {}^q\mathbb{R}_q^{\mathbb{U}} = O_p \times O_q$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}}$$

II2 prod good

$$Z_{\mathbb{C}} = {}^n\mathbb{C}_n^{\mathfrak{D}} \times {}^n\mathbb{C}_n^{\mathfrak{D}} \supset B_{\mathbb{C}}$$

$$Z_{\mathbb{R}} = {}^n\mathbb{C}_n^{\mathfrak{D}} \text{ real} \supset B_{\mathbb{R}}$$

$$G_{\mathbb{C}} = {}^{2n}\mathbb{R}_{2n}^{\Omega} \times {}^{2n}\mathbb{R}_{2n}^{\Omega} = Sp_n(\mathbb{R}) \times Sp_n(\mathbb{R})$$

$$G_{\mathbb{R}} = {}^{2n}\mathbb{R}_{2n}^{\Omega} \text{ diag} = Sp_n(\mathbb{R})$$

$$G_{\mathbb{C}}^c = {}^{2n}\mathbb{C}_{2n}^{\Omega} = Sp_n(\mathbb{C})$$

$$K_{\mathbb{C}} = {}^n\mathbb{C}_n^{\mathfrak{U}} \times {}^n\mathbb{C}_n^{\mathfrak{U}} = U_n \times U_n$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}} = {}^n\mathbb{C}_n^{\mathfrak{U}} = U_n \text{ diag}$$

III0 Cau

$$Z_{\mathbb{C}} \supset B_{\mathbb{C}}$$

$$Z_{\mathbb{R}} = {}^n\mathbb{H}_n^{\mathfrak{U}} \supset B_{\mathbb{R}}$$

$$G_{\mathbb{C}} = G_{\mathbb{C}}^c = {}^{2n}\mathbb{H}_{2n}^{\mathfrak{D}} = SO_{2n}^*$$

$$G_{\mathbb{R}} = {}^n\mathbb{H}_n^{\mathfrak{C}} \times \mathbb{R}^{\times} = SL_n(\mathbb{H}) \times \mathbb{R}^{\times} = SU_{2n}^* \times \mathbb{R}^{\times}$$

$$K_{\mathbb{C}} = {}^{2n}\mathbb{C}_{2n}^{\mathfrak{U}} = U_{2n}$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}} = {}^{2n}\mathbb{C}_{2n}^{\Omega*} = {}^{2n}\mathbb{C}_{2n}^{\Omega} \cap {}^{2n}\mathbb{C}_{2n}^{\mathfrak{U}} = Sp_n$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}}$$

III1

$$Z_{\mathbb{C}} \supset B_{\mathbb{C}}$$

$$Z_{\mathbb{R}} = {}^p\mathbb{H}_q \supset B_{\mathbb{R}}$$

$$G_{\mathbb{C}} = {}^{2p:2q}\mathbb{C}_{2p:2q}^{\mathfrak{U}} = U_{2p:2q}$$

$$G_{\mathbb{R}} = {}^{2p:2q}\mathbb{C}_{2p:2q}^{\mathfrak{U}} \cap {}^{2(p+q)}\mathbb{C}_{2(p+q)}^{\Omega} = Sp_{p:q}$$

$$G_{\mathbb{C}}^c = {}^{p+q}\mathbb{H}_{p+q} = SU_{2(p+q)}^*$$

$$K_{\mathbb{C}} = {}^{2p}\mathbb{C}_{2p}^{\mathfrak{U}} \times {}^{2q}\mathbb{C}_{2q}^{\mathfrak{U}} = U_{2p} \times U_{2q}$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}} = {}^p\mathbb{H}_p^{\cup} \times {}^q\mathbb{H}_q^{\cup} = Sp_p \times Sp_q$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}}$$

III2 prod

$$Z_{\mathbb{C}} = {}^n\mathbb{C}_n^{\ni} \times {}^n\mathbb{C}_n^{\ni} \supset B_{\mathbb{C}}$$

$$Z_{\mathbb{R}} = {}^n\mathbb{C}_n^{\ni} \text{ real} \supset B_{\mathbb{R}}$$

$$G_{\mathbb{C}} = {}^n\mathbb{H}_n^{\ni} \times {}^n\mathbb{H}_n^{\ni} = SO_{2n}^* \times SO_{2n}^*$$

$$G_{\mathbb{R}} = {}^n\mathbb{H}_n^{\ni} = SO_{2n}^* \text{ diag}$$

$$G_{\mathbb{C}}^c = {}^{2n}\mathbb{C}_{2n}^{\ni} = SO_{2n}(\mathbb{C})$$

$$K_{\mathbb{C}} = {}^n\mathbb{C}_n^{\cup} \times {}^n\mathbb{C}_n^{\cup} = U_n \times U_n$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}} = {}^n\mathbb{C}_n^{\cup} = U_n \text{ diag}$$

IV0

$$Z_{\mathbb{C}} \supset B_{\mathbb{C}}$$

$$Z_{\mathbb{R}} = \mathbb{R} \times \mathbb{R}^{n-1} = \mathbb{R}^{1:n-1} \supset B_{\mathbb{R}}$$

$$G_{\mathbb{C}} = G_{\mathbb{C}}^c = {}_{2:n}^{\ni}\mathbb{R}^{2:n} = SO_{2:n}$$

$$G_{\mathbb{R}} = {}_{1:n-1}^{\ni}\mathbb{R}^{1:n-1} \times \mathbb{R}^{\times} = SO_{1:n-1} \times \mathbb{R}^{\times}$$

$$K_{\mathbb{C}}$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}}$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}}$$

IV1

$$G_{\mathbb{C}} = {}_n^{\ni}\mathbb{R}^n \times \mathbb{T} = SO_n \times \mathbb{T}$$

$$G_{\mathbb{R}} = {}_{n-1}^{\ni}\mathbb{R}^{n-1} = SO_{n-1}$$

IV:2

$$Z_{\mathbb{C}} \supset B_{\mathbb{C}}$$

$$Z_{\mathbb{R}} = \mathbb{R}^{n-p} \Rightarrow B_{\mathbb{R}}$$

$$G_{\mathbb{C}} = {}_p^{\ni}\mathbb{R}^p \times {}_{2:n-p}^{\ni}\mathbb{R}^{2:n-p} = SO_p \times SO_{2:n-p}$$

$$G_{\mathbb{R}} = {}_{p-1} \circlearrowleft \mathbb{R}^{p-1} \times {}_{1:n-p} \circlearrowleft \mathbb{R}^{1:n-p} = SO_{p-1} \times SO_{1:n-p}$$

$$K_{\mathbb{C}}$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}}$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}}$$

IV3

$$Z_{\mathbb{C}} \supset B_{\mathbb{C}}$$

$$Z_{\mathbb{R}} = \mathbb{R}^{n-1} \Rightarrow B_{\mathbb{R}}$$

$$G_{\mathbb{C}} = {}_{2:n-1} \circlearrowleft \mathbb{R}^{2:n-1} = SO_{2:n-1}$$

$$G_{\mathbb{R}} = {}_{1:n-1} \circlearrowleft \mathbb{R}^{1:n-1} = SO_{1:n-1}$$

$$K_{\mathbb{C}}$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}}$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}}$$

IV prod

$$Z_{\mathbb{C}} = \mathbb{C}^n \times \mathbb{C}^n \supset B_{\mathbb{C}}$$

$$Z_{\mathbb{R}} = \mathbb{C}^n \text{ real} \Rightarrow B_{\mathbb{R}}$$

$$G_{\mathbb{C}} = {}_{2:n} \circlearrowleft \mathbb{R}^{2:n} \times {}_{2:n} \circlearrowleft \mathbb{R}^{2:n} = SO_{2:n} \times SO_{2:n}$$

$$G_{\mathbb{R}} = {}_{2:n} \circlearrowleft \mathbb{R}^{2:n} = SO_{2:n}$$

$$G_{\mathbb{C}}^c = {}_{2+n} \circlearrowleft \mathbb{C}^{2+n} = SO_{2+n} (\mathbb{C})$$

$$K_{\mathbb{C}}$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}}$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}}$$

IV5

$$G_{\mathbb{C}} = {}_{2:p+q} \circlearrowleft \mathbb{R}^{2:p+q} = SO_{2:p+q}$$

$$G_{\mathbb{R}} = {}_{1:p} \circlearrowleft \mathbb{R}^{1:p} \times {}_{1:q} \circlearrowleft \mathbb{R}^{1:q} = SO_{1:p} \times SO_{1:q}$$

$$G_{\mathbb{C}}^c = {}_{p+1:q+1} \circlearrowleft \mathbb{R}^{p+1:q+1} = SO_{p+1:q+1}$$

$$K_{\mathbb{C}}$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}}$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}}$$

V1 cpt

$$G_{\mathbb{C}} = E_{6: -14} \times \mathbb{T}$$

$$G_{\mathbb{R}} = F_{4: -20}$$

$$G_{\mathbb{C}}^c = E_{6: -26}$$

$$K_{\mathbb{C}}$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}}$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}}$$

$$S = \text{cpt}$$

V2

$$G_{\mathbb{C}} = {}^U_{6:2}\mathbb{C}^{6:2} = SU_{6:2}$$

$$G_{\mathbb{R}} = {}^U_{6:2}\mathbb{C}^{6:2} \cap {}^N_8\mathbb{C}^8 = Sp_{3:1}$$

$$G_{\mathbb{C}}^c$$

$$K_{\mathbb{C}}$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}}$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}}$$

V prod

$$Z_{\mathbb{C}} = \mathbb{O}_{\mathbb{C}}^{1 \times 2} \times \mathbb{O}_{\mathbb{C}}^{1 \times 2}$$

$$Z_{\mathbb{R}} = \mathbb{O}_{\mathbb{C}}^{1 \times 2} \text{ real}$$

$$G_{\mathbb{C}} = E_{6: -14} \times E_{6: -14}$$

$$G_{\mathbb{R}} = E_{6: -14} \text{ diag}$$

$$G_{\mathbb{C}}^c = E_6$$

$$K_{\mathbb{C}}$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}}$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}}$$

VI0 Cau

$$Z_{\mathbb{C}}$$

$$Z_{\mathbb{R}} = {}^3\mathbb{O}_3^{\mathfrak{U}}$$

$$G_{\mathbb{C}} = E_{7:-25} = G_{\mathbb{C}}^c$$

$$G_{\mathbb{R}} = E_{6:-26} \times \mathbb{R}^{\times}$$

$$K_{\mathbb{C}}$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}}$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}}$$

VI prod

$$Z_{\mathbb{C}} = {}^3\mathbb{O}_3^{\mathfrak{U}} \times {}^3\mathbb{O}_3^{\mathfrak{U}}$$

$$Z_{\mathbb{R}} = {}^3\mathbb{O}_3^{\mathfrak{U}} \text{ real}$$

$$G_{\mathbb{C}} = E_{7:-25} \times E_{7:-25}$$

$$G_{\mathbb{R}} = E_{7:-25} \text{ diag}$$

$$G_{\mathbb{C}}^c = E_7$$

$$K_{\mathbb{C}}$$

$$K_{\mathbb{R}} = K_{\mathbb{C}} \cap G_{\mathbb{R}}$$

$$K_{\mathbb{R}} : \times K_{\mathbb{C}}$$

VI3

$$G_{\mathbb{C}} = E_{7:-25}$$

$$G_{\mathbb{R}} = {}^4\mathbb{H}_4^{\mathbb{C}} = SU_8^*$$

$$G_{\mathbb{C}}^c = E_{7:7}$$