$$\mathbb{N}\,\triangleleft^{0}_{\mathbb{P}}$$

$$\Big\downarrow$$

$$\mathbb{N}^{\times}$$

$$\mathbb{N}\,\triangleleft^{0}_{\mathbb{P}} \ni \mu \mapsto \prod_{p}^{\mathbb{P}} p^{\mu_p} = \bcancel{\mu} \in \mathbb{N}^{\times} : \quad \bcancel{\nu} = \prod_{p}^{\mathbb{P}} p^{\nu_p} = p_1^{\nu_1} \cdots p_k^{\nu_k} \in \mathbb{N}^{\times}$$

$$\nu = \xcancel{\bcancel{\nu}}$$

$$\bigwedge_{n}^{\mathbb{N}^{\times}} \bigvee_{\nu} n = \bcancel{\nu}$$

$$\text{if } n \in \mathbb{P} \Rightarrow n = \bcancel{\chi}^{\,n}$$

$$\text{if } n \notin \mathbb{P} \Rightarrow \bigvee_{\dot{m}}^{\mathbb{N}^{\times}} n = m\acute{m} \ \underset{\text{ind}}{\Longrightarrow} \ \bigvee_{\dot{\mu}} \dot{m} = \bcancel{\dot{\mu}} \Rightarrow n = \bcancel{\mu}\,\bcancel{\acute{\mu}} = \mu \bcancel{+} \acute{\mu}$$

$$\bcancel{\mu} = \bcancel{\nu} \underset{\text{inj}}{\Longrightarrow} \mu = \nu$$

$$N = \begin{cases} n \in \mathbb{N}^{\times} \\ \bigvee\limits_{\nu}^{\text{eind}} \bcancel{\nu} = n \end{cases}$$

$$\bcancel{\mu} = 1 \Longrightarrow \mu = 0 \Longrightarrow 1 \in N$$

$$p \prec \bcancel{\nu} \in N \Longrightarrow \nu_p \geqslant 1$$

$$\bcancel{\nu}/p = \bcancel{\mu} \Longrightarrow \bcancel{\nu} = p\bcancel{\mu} = \bcancel{\chi^p}\,\bcancel{\mu} = \bcancel{\chi^p + \mu} \underset{\text{eind}}{\Longrightarrow} \nu = \chi^p + \mu \Longrightarrow \nu_p = 1 + \mu_p \geqslant 1$$

$$\lightning \bigvee_{\mu \neq \nu} \bcancel{\mu} = \bcancel{\nu} > 1$$

$$\text{Trg}\ \mu \cap \text{Trg}\ \nu = \varnothing$$

$$\lightning \bigvee_{q}^{\mathbb{P}} \mu_q \geqslant 1 \leqslant \nu_q \Longrightarrow \bcancel{\mu - \chi^q} = \bcancel{\mu}/q = \bcancel{\nu}/q = \bcancel{\nu - \chi^q} \underset{\text{ind}}{\Longrightarrow} \mu - \chi^q = \nu - \chi^q \Longrightarrow \mu = \nu$$

$$p = \min\ \text{Trg}\ \mu \underset{\text{OE}}{<} q = \min\ \text{Trg}\ \nu$$

$$\bcancel{\nu} > pq$$

$$\lightning\ \nu = \chi^q \Longrightarrow q = \bcancel{\nu} = \bcancel{\mu} \succ p\ \lightning \Longrightarrow \nu \neq \chi^q \Longrightarrow \begin{cases} \nu_q > 1 & \Longrightarrow \bcancel{\nu} \geqslant q^2 > pq \\ \nu_q = 1 & \Longrightarrow \bigvee\limits_{q < \acute{q}}^{\mathbb{P}} \nu_{\acute{q}} \geqslant 1 \Longrightarrow \bcancel{\nu} \geqslant q\acute{q} > q^2 > pq \end{cases}$$

$$p \prec \bcancel{\nu} \succ q \Longrightarrow p \prec \bcancel{\nu} - pq = \bcancel{\lambda} \succ q \underset{\bcancel{\nu} > \bcancel{\lambda} \in N}{\overset{\text{ind}}{\Longrightarrow}} \lambda_p \geqslant 1 \leqslant \lambda_q \underset{p \neq q}{\Longrightarrow} pq \prec \bcancel{\lambda}$$

$$\Longrightarrow pq \prec \bcancel{\nu} \Longrightarrow p \prec \bcancel{\nu}/q = \bcancel{\nu - \chi^q} \underset{\bcancel{\nu} > \bcancel{\nu - \chi^q} \in N}{\overset{\text{ind}}{\Longrightarrow}} p \in \text{Trg}\ \underbrace{\nu - \chi^q} \geqslant q\ \lightning$$

$$\mu \leqslant \nu \Longrightarrow \bcancel{\mu} \prec \bcancel{\nu}$$

$$\mu \leqslant \nu \Longrightarrow \bigwedge_{p}^{\mathbb{P}} \nu_p - \mu_p \geqslant 0 \Longrightarrow \bcancel{\nu} = \prod_{p}^{\mathbb{P}} p^{\nu_p} = \bcancel{\nu} = \prod_{p}^{\mathbb{P}} p^{\nu_p - \mu_p} \prod_{p}^{\mathbb{P}} p^{\mu_p} = \prod_{p}^{\mathbb{P}} p^{\nu_p - \mu_p} \bcancel{\mu} \Longrightarrow \bcancel{\mu} \prec \bcancel{\nu}$$