

$$\mathcal{V}^1 \cdots \mathcal{V}^n \in R$$

$$R^n = \frac{(\mathcal{V}^1 \cdots \mathcal{V}^n)}{\mathcal{V}^i \in R} \text{ free Mod}$$

$$\overset{k}{\mathfrak{X}} R^n = \frac{(\mathcal{V}^I : |I| = k)}{\mathcal{V}^I \in R}$$

$$R \sqsupset R \langle \mathcal{V}^1 \cdots \mathcal{V}^n \rangle \xleftarrow{\partial_{\mathcal{V}}^0} \overset{0}{\mathfrak{X}} R^n \xleftarrow{\partial_{\mathcal{V}}^1} \overset{1}{\mathfrak{X}} R^n \xleftarrow{\partial_{\mathcal{V}}^k} \overset{k}{\mathfrak{X}} R^n \xleftarrow{\partial_{\mathcal{V}}^{k+1}} \overset{k+1}{\mathfrak{X}} R^n \xleftarrow{\partial_{\mathcal{V}}^{n-1}} \overset{n-1}{\mathfrak{X}} R^n \xleftarrow{\partial_{\mathcal{V}}^n} \overset{n}{\mathfrak{X}} R^n$$

$$\mathcal{V} \overleftarrow{\overset{k+1}{\mathfrak{X}} R^n}^I = \overleftarrow{\partial_{\mathcal{V}}(\mathcal{V}^J)}_{|J|=k+1}^I = \sum_j^{N-I} \mathcal{V}^j \mathcal{V}^{I \cup j}$$

$$\partial_{\mathcal{V}}(\mathcal{V}^j)_{j \in N} = \sum_j^N \mathcal{V}^j \quad \mathcal{V}^j \in \langle \mathcal{V}^1 \cdots \mathcal{V}^n \rangle R$$

$$\langle \mathcal{V}^1 \cdots \mathcal{V}^n \rangle R \supset \langle \mathcal{V}^1 \cdots \mathcal{V}^n \rangle R$$

$$\begin{array}{ccccccc} R \sqsupset \langle \mathcal{V}^1 \cdots \mathcal{V}^n \rangle R & \xleftarrow{\partial_{\mathcal{V}}^0} & \overset{0}{\mathfrak{X}} R^n & \xleftarrow{\partial_{\mathcal{V}}^1} & \overset{1}{\mathfrak{X}} R^n & \xleftarrow{\partial_{\mathcal{V}}^k} & \overset{k}{\mathfrak{X}} R^n & \xleftarrow{\partial_{\mathcal{V}}^n} & \overset{n}{\mathfrak{X}} R^n \\ \downarrow & & \downarrow \overset{0}{\mathfrak{X}} \mathcal{A} & & \downarrow \overset{1}{\mathfrak{X}} \mathcal{A} & & \downarrow \overset{k}{\mathfrak{X}} \mathcal{A} & & \downarrow \overset{n}{\mathfrak{X}} \mathcal{A} \\ R \sqsupset \langle \mathcal{V}^1 \cdots \mathcal{V}^n \rangle R & \xleftarrow{\partial_{\mathcal{V}}^0} & \overset{0}{\mathfrak{X}} R^n & \xleftarrow{\partial_{\mathcal{V}}^1} & \overset{1}{\mathfrak{X}} R^n & \xleftarrow{\partial_{\mathcal{V}}^k} & \overset{k}{\mathfrak{X}} R^n & \xleftarrow{\partial_{\mathcal{V}}^n} & \overset{n}{\mathfrak{X}} R^n \end{array}$$

$$\mathcal{V}^i = \mathcal{V}^j \mathcal{A}_i^j$$

$$\overset{k}{\mathfrak{X}} R^n \leftarrow \overset{k}{\mathfrak{X}} R^n$$

$$\overleftarrow{\overset{k}{\mathfrak{X}} \mathcal{A}(\mathcal{V}^J : |J| = k)}^I = \mathcal{V}^J \overleftarrow{\mathcal{A}^I}^I$$

$$\overleftarrow{\overset{1}{\mathfrak{X}} \mathcal{A}(\mathcal{V}^j)}^i = \mathcal{V}^j \mathcal{A}_j^i$$