$$
\begin{array}{ccccccc}
\llcorner_{\triangle}^{-}\mathbb{1}^{\mathbb{N}} & \ni & \urcorner = \nu^{I}\,\overbrace{{}_{I}\llcorner\urcorner} & \llcorner_{\triangle}^{-}\mathbb{1}^{\mathbb{N}} & \ni & \urcorner = \nu^{I}\,\overbrace{{}_{I}\widetilde{\llcorner\urcorner}} \\
\downarrow{\scriptstyle \bullet\llcorner} & & \downarrow & \downarrow{\scriptstyle \bullet\llcorner} & & \downarrow \\
{}_{2^{Q}}\mathbb{1} & \ni & {}_{I}\llcorner\urcorner & {}_{2^{Q}}\mathbb{1} & \ni & {}_{I}\llcorner\urcorner
\end{array}
$$

$$\llcorner_{\triangle}^{-}\mathbb{1}^{\mathbb{N}} = \llcorner_{\triangle}^{-}\mathbb{K}^{\mathbb{N}} \bowtie \mathbb{1}$$

$$\nu^{1} \cdots \nu^{n} \in \llcorner_{\triangle}^{-}\mathbb{1}^{\geqslant 1} \text{ nilpotent}$$

$$\nu^{J} = \llcorner^{I} \; {}_{I}\nu^{J}$$

$$\nu^{j} = \llcorner^{I} \; {}_{I}\nu^{j} = \llcorner^{i} \; {}_{i}\nu^{j} + \sum_{|I| > 1} \llcorner^{I} \; {}_{I}\nu^{j} = \llcorner^{i} \; {}_{i}\nu^{j} + \overset{>}{\nu}^{j}$$

$$\underset{-}{\deg}\left(\nu^{j} - \llcorner^{i} \, {}_{i}\nu^{j}\right) = \underset{-}{\deg}\, \overset{>}{\nu}^{j} \geqslant 2$$

$${}_{i}\nu^{j} \in \mathbb{1}_{0}$$

$$(1)\ \det \; {}_{\cdot}\nu^{\cdot} \in \mathbb{1}_{0}^{\mathsf{C}} \text{ inv}$$

$${}_{\cdot}\nu^{\cdot} \in {}_{p}^{\mathsf{C}}\mathbb{1}_{0}^{p} \text{ inv}$$

$${}_{\cdot}\llcorner^{\cdot} := {}_{\cdot}\nu^{-1}$$

$$\llcorner^{i} = \left(\nu^{j} - \overset{>}{\nu}^{j}\right) {}_{j}\llcorner^{i} = \nu^{j} \; {}_{j}\llcorner^{i} - \overset{>}{\nu}^{j} \; {}_{j}\llcorner^{i} = \nu^{j} \; {}_{j}\llcorner^{i} + \underbrace{\overset{>}{\llcorner}^{i}}_{\geqslant 2}$$

$$(2)\ \llcorner_{\triangle}^{-}\mathbb{1}^{\mathbb{N}} = [\,\nu^{1} \cdots \nu^{n}\,]\,\mathbb{1}_{\text{free}} \quad \text{Algebra } \mathbb{1}$$

$$\llcorner_{\triangle}^{-}\mathbb{1}^{\mathbb{N}} \xleftarrow[\text{bij}]{\text{hom } \mathbb{1}} \llcorner_{\triangle}^{-}\mathbb{1}^{\mathbb{N}}$$

$$\nu^{I}\,\urcorner \hookleftarrow \llcorner^{I} \bowtie \urcorner$$

$$(3)\ \llcorner_{\triangle}^{-}\mathbb{1}^{\mathbb{N}} = [\,\nu^{1} \cdots \nu^{n}\,]\,\mathbb{1} \quad \text{Algebra } \mathbb{1}$$

$$\llcorner_{\triangle}^{-}\mathbb{1}^{\mathbb{N}} \xleftarrow[\text{surj}]{\text{hom } \mathbb{1}} \llcorner_{\triangle}^{-}\mathbb{1}^{\mathbb{N}}$$

$$\ldots^{I} \ldots \ldots^{I} \ldots$$

$$(4)\quad \ldots^{-\,\geqslant 1} =< \ldots^{1} \cdots \ldots^{n} > \ldots^{-\,\mathbb{N}} \quad \text{ideal}$$

$$(5)\quad \ldots^{-\,\geqslant 1} + \underbrace{\ldots^{-\,\geqslant 2}} =< \ldots^{i} + \ldots^{-\,\geqslant 2} > \ldots \quad \text{Module } \ldots$$

$$(6)\quad \ldots^{-\,\geqslant n} =< \ldots^{1} \cdots \ldots^{n} > \ldots^{-\,\mathbb{N}} \quad \text{ideal}$$

$$\ldots^{1} \cdots \ldots^{n} = \ldots^{1} \cdots\ldots^{n} \det \ldots$$

$$\ldots^{N} = \ldots^{N} \det \ldots$$

$$\ldots^{1} \cdots \ldots^{n} = \overbrace{\ldots^{j_1}{}_{j_1}\ldots^{1} + \ldots^{>1}} \cdots \overbrace{\ldots^{j_n}{}_{j_n}\ldots^{n} + \ldots^{>n}} = \overbrace{\ldots^{j_1}{}_{j_1}\ldots^{1}} \cdots \overbrace{\ldots^{j_n}{}_{j_n}\ldots^{n}}$$

$$+ \underbrace{\overbrace{\ldots^{j_1}{}_{j_1}\ldots^{1}} \cdots \ldots^{>i_1} \cdots \overbrace{\ldots^{j_n}{}_{j_n}\ldots^{n}}}_{\deg \geqslant n+1} + \underbrace{\overbrace{\ldots^{j_1}{}_{j_1}\ldots^{1}} \cdots \ldots^{>i_1} \cdots \ldots^{>i_2} \cdots \overbrace{\ldots^{j_n}{}_{j_n}\ldots^{n}}}_{\deg \geqslant n+2} + \cdots + \underbrace{\overbrace{\ldots^{j_1}{}_{j_1}\ldots^{1} + \ldots^{>1}} \cdots \overbrace{\ldots^{j_n}{}_{j_n}\ldots^{n} + \ldots^{>n}}}_{\deg \geqslant 2n}$$

$$= \overbrace{\ldots^{j_1}{}_{j_1}\ldots^{1}} \cdots \overbrace{\ldots^{j_n}{}_{j_n}\ldots^{n}} = \ldots^{1} \cdots \ldots^{n} \det \ldots$$

$$(1) \Longrightarrow (2)$$

surj

$$\text{Ind} \bigwedge_{0 \leqslant k} \llcorner^{i} = \sum_{|J| < k} \lrcorner^{J} \;{}_{J}\llcorner^{i} + \underbrace{\overset{>}{\llcorner}^{i}}_{\geqslant k}$$

$$k = 0 \text{ klar}$$

$$0 \leqslant k \curvearrowright j+1: \quad \text{Vor} \quad \llcorner^{i} = \sum_{|J| < k} \lrcorner^{J} \;{}_{J}\llcorner^{i} + \sum_{|J| \geqslant k} \llcorner^{J} \;{}_{J}\urcorner^{i}$$

$$\Longrightarrow \llcorner^{i} - \sum_{|J| < k} \lrcorner^{J} \;{}_{J}\llcorner^{i} = \sum_{|J| \geqslant k} \llcorner^{J} \;{}_{J}\urcorner^{i} = \sum_{|J| = k} \llcorner^{J} \;{}_{J}\urcorner^{i} + \sum_{|J| > k} \llcorner^{J} \;{}_{J}\urcorner^{i}$$

$$\Longrightarrow \llcorner^{i} - \sum_{|J| < k} \lrcorner^{J} \;{}_{J}\llcorner^{i} - \sum_{|J| > k} \llcorner^{J} \;{}_{J}\urcorner^{i} = \sum_{|J| = k} \llcorner^{J} \;{}_{J}\urcorner^{i} = \sum_{|J| = k} \prod_{j}^{J} \llcorner^{j} \;{}_{J}\urcorner^{i}$$

$$= \sum_{|J| = k} \prod_{j}^{J} \underbrace{\lrcorner^{\ell} \;{}_{\ell}\llcorner^{j} + \overset{>}{\llcorner}^{j}} \;{}_{J}\urcorner^{i} = \sum_{|J| = k} \underbrace{\lrcorner^{\ell_1} \;{}_{\ell_1}\llcorner^{j_1} + \overset{>}{\llcorner}^{j_1}} \cdots \underbrace{\lrcorner^{\ell_k} \;{}_{\ell_k}\llcorner^{j_k} + \overset{>}{\llcorner}^{j_k}} \;{}_{J}\urcorner^{i}$$

$$= \sum_{|J| = k} \left(\lrcorner^{\ell_1} \;{}_{\ell_1}\llcorner^{j_1} \cdots \lrcorner^{\ell_k} \;{}_{\ell_k}\llcorner^{j_k} + \underbrace{\lrcorner^{\ell_1} \;{}_{\ell_1}\llcorner^{j_1} \cdots \overset{>}{\llcorner}^{i_1} \cdots \lrcorner^{\ell_k} \;{}_{\ell_k}\llcorner^{j_k}}_{\geqslant k-1+2 = k+1} + \underbrace{\lrcorner^{\ell_1} \;{}_{\ell_1}\llcorner^{j_1} \cdots \overset{>}{\llcorner}^{i_1} \cdots \overset{>}{\llcorner}^{i_2} \cdots \lrcorner^{\ell_k} \;{}_{\ell_k}\llcorner^{j_k}}_{\geqslant k-2+2 \cdot k+2} + \cdots + \underbrace{\overset{>}{\llcorner}^{j_1} \cdots \overset{>}{\llcorner}^{j_k}}_{\geqslant 2k} \right) \;{}_{J}\urcorner^{i}$$

$$= \sum_{|J| = k} \prod_{j}^{J} \lrcorner^{\ell} \;{}_{\ell}\llcorner^{j} \;{}_{J}\urcorner^{i} + \underbrace{\urcorner}_{> k} = \sum_{|L| = k} \lrcorner^{L} \;{}_{L}\llcorner^{i} + \urcorner$$

$$\text{inj} \quad \llcorner^{-}_{\triangle}\overset{\mathbb{N}}{\mathbb{K}} \boxtimes \mathbb{1} \ni \urcorner = \sum_{I} \llcorner^{I} \;\boxtimes_{I}\urcorner \in \ker\varphi \Longrightarrow \sum_{J} \lrcorner^{J} \;{}_{J}\urcorner = 0$$

$$\lightning \; \urcorner \neq 0 \Longrightarrow \underline{\deg}\, \urcorner = k \geqslant 0 \Longrightarrow \urcorner = \sum_{|J| \geqslant k} \llcorner^{J} \;{}_{J}\urcorner$$

$$\bigvee_{I} \begin{cases} |I| = k \\ {}_{I}\urcorner \neq 0 \end{cases}$$

$$|J| \geqslant k$$

$$J \neq I \Longrightarrow \underbrace{N \llcorner I} \cap J \neq \varnothing \Longleftarrow \underbrace{N \llcorner I} \cap J = \varnothing \Longrightarrow N \llcorner I \subset N \llcorner J \Longrightarrow J \subset I \underset{|J| \geqslant k}{\Longrightarrow} J = I$$

$$0 = \lrcorner^{N \llcorner I} \sum_{J} \lrcorner^{J} \;{}_{J}\urcorner = \sum_{J} \lrcorner^{N \llcorner I} \;\lrcorner^{J} \;{}_{J}\urcorner = \lrcorner^{N \llcorner I} \;\lrcorner^{I} \;{}_{I}\urcorner = \pm \lrcorner^{N} \;{}_{I}\urcorner = \pm \;\llcorner^{N} \underbrace{\det \lrcorner}_{\text{inv}} {}_{I}\urcorner \neq 0 \;\lightning$$

(2) ⇒ (3)

klar

(3) ⇒ (4)

$$\text{surj} \Rightarrow \mathtt{L}^i = \mathtt{V}^J\ {}_J\mathtt{A}^i = \prod_j^J \mathtt{V}^j\ {}_J\mathtt{A}^i = {}_\varnothing\mathtt{A}^i + \sum_{\varnothing \neq J} \underbrace{\prod_j^J \mathtt{V}^j}_{\geqslant |J| > 0}\ {}_J\mathtt{A}^i \Rightarrow {}_\varnothing\mathtt{A}^i = 0$$

$$\Rightarrow \mathtt{L}^i = \sum_{\varnothing \neq J} \prod_j^J \mathtt{V}^j\ {}_J\mathtt{A}^i \in < \mathtt{V}^1 \cdots \mathtt{V}^n > {}^{\mathtt{L}}\triangle^{-}\mathbb{1}^{\mathbb{N}}$$

$$\Rightarrow {}^{\mathtt{L}}\triangle^{-}\mathbb{1}^{\geqslant 1} = < \mathtt{L}^1 \cdots \mathtt{L}^n > {}^{\mathtt{L}}\triangle^{-}\mathbb{1}^{\mathbb{N}} \sqsubset < \mathtt{V}^1 \cdots \mathtt{V}^n > {}^{\mathtt{L}}\triangle^{-}\mathbb{1}^{\mathbb{N}}$$

(4) ⇒ (5)

$$\mathtt{L}^i = \mathtt{V}^j\ {}_j\mathbb{1}^i$$

$${}^{\mathtt{L}}\triangle^{-}\mathbb{1}^{\mathbb{N}} \ni {}_j\mathbb{1}^i = \underbrace{{}_j\mathbb{1}^i_0}_{\in \mathbb{1}} + \underbrace{{}_j\mathbb{1}^i_>}_{\geqslant 1}$$

$$\Rightarrow \mathtt{L}^i = \mathtt{V}^j \underbrace{{}_j\mathbb{1}^i_0 + {}_j\mathbb{1}^i_>} = \mathtt{V}^j\ {}_j\mathbb{1}^i_0 + \underbrace{\mathtt{V}^j\ {}_j\mathbb{1}^i_>}_{\geqslant 2}$$

$$\Rightarrow \mathtt{L}^i + {}^{\mathtt{L}}\triangle^{-}\mathbb{1}^{\geqslant 2} = \mathtt{V}^j\ {}_j\mathbb{1}^i_0 + {}^{\mathtt{L}}\triangle^{-}\mathbb{1}^{\geqslant 2} \in < \mathtt{V}^j + {}^{\mathtt{L}}\triangle^{-}\mathbb{1}^{\geqslant 2} > \mathbb{1}$$

$$(5) \Longrightarrow (6)$$

$$\lrcorner^i = \llcorner^j\ {}_j\lrcorner^i + \tilde{\lrcorner}^i$$

$${}_j\lrcorner^i \in \mathbb{1}_0$$

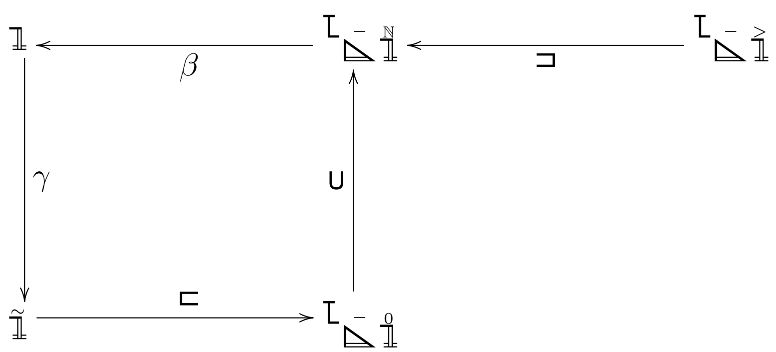
$$\tilde{\lrcorner}^i \in \lrcorner_{\triangle}^{-\ \geqslant 2}\mathbb{1} \underset{\llcorner^i\ \mathrm{odd}}{\Longrightarrow} \lrcorner^1 \cdots \lrcorner^n = \det \llcorner^{\cdot}\ \llcorner^1 \cdots \llcorner^n$$

$$\Longrightarrow \lrcorner_{\triangle}^{-\ \geqslant n}\mathbb{1} = < \lrcorner^1 \cdots \lrcorner^n > \lrcorner_{\triangle}^{-\ \mathbb{N}}\mathbb{1} \sqsubset < \llcorner^1 \cdots \llcorner^n > \lrcorner_{\triangle}^{-\ \mathbb{N}}\mathbb{1}$$

$$(6) \Longrightarrow (1)$$

$$0 \neq \lrcorner^1 \cdots \lrcorner^n = \llcorner^1 \cdots \llcorner^n \underbrace{\mathbb{1}}_{\in \mathbb{1}_0} = \underbrace{\lrcorner^1 \cdots \lrcorner^n \det \llcorner^{\cdot}}\mathbb{1} = \lrcorner^1 \cdots \lrcorner^n \underbrace{\det \llcorner^{\cdot}\mathbb{1}}$$

$$\Longrightarrow \det \llcorner^{\cdot}\mathbb{1} = 1 \Longrightarrow \det \llcorner^{\cdot} \in \mathbb{1}_0^{\mathsf{c}}\ \mathrm{inv}$$



$${}^{\gamma}\mathbb{1} = \tilde{\mathbb{1}} \in \tilde{\mathbb{1}} \sqsubset \lrcorner_{\triangle}^{-\ 0}\mathbb{1}\ \text{Sub-Alg}$$

$$\tilde{\mathbb{1}}_{\varnothing} = \mathbb{1}$$

$$\det \llcorner^{\cdot} \in \mathbb{1}_0^{\mathsf{c}}\ \mathrm{inv}$$

$$\lrcorner_{\triangle}^{-\ \mathbb{N}}\mathbb{1} = [\llcorner^1 \cdots \llcorner^n]\,\tilde{\mathbb{1}}_{\text{free}} \quad \text{Algebra } \mathbb{1}$$

$$\lrcorner_{\triangle}^{-\ \mathbb{N}}\mathbb{1} \xleftarrow[\text{bij}]{\text{hom } \mathbb{1}} \lrcorner_{\triangle}^{-\ \mathbb{N}}\mathbb{1}$$

$$\lrcorner^{I}\ \tilde{\mathbb{1}} \hookleftarrow \llcorner^{I}\ \boxtimes \mathbb{1}$$

surj

$$\text{Ind} \bigwedge_{0 \leqslant k} \mathbb{1} = \sum_{|J| < k} \lrcorner^{J}\ {}_{J}\tilde{\mathbb{1}} + \sum_{|J| \geqslant k} \lrcorner^{J}\ \mathbb{1}_{J}$$

$k = 0$ klar

$$0 \leqslant k \lightning j+1\text{: Vor } \mathbb{1} - \sum_{|J| < k} \lrcorner^{J}\ {}_{J}\tilde{\mathbb{1}} = \sum_{|J| \geqslant k} \lrcorner^{J}\ \mathbb{1}_{J}$$

$$\bigwedge_{|J| \geqslant k} \bigvee_{{}_{J}\mathbb{1} \in \mathbb{1}} \mathbb{1}_{J} = {}_{J}\tilde{\mathbb{1}}_{\varnothing}$$

$$\Rightarrow \mathbb{1} - \sum_{|J| \leqslant k} \lrcorner^{J}\ {}_{J}\tilde{\mathbb{1}} = \mathbb{1} - \sum_{|J| < k} \lrcorner^{J}\ {}_{J}\tilde{\mathbb{1}} - \sum_{|J| = k} \lrcorner^{J}\ {}_{J}\tilde{\mathbb{1}}$$

$$= \sum_{|J| \geqslant k} \lrcorner^{J}\ {}_{J}\tilde{\mathbb{1}}_{\varnothing} - \sum_{|J| = k} \lrcorner^{J}\ {}_{J}\tilde{\mathbb{1}} = \underbrace{\sum_{|J| = k} \overset{>k}{\lrcorner^{J}}\ \underbrace{{}_{J}\tilde{\mathbb{1}}_{\varnothing} - {}_{J}\tilde{\mathbb{1}}}_{>1}} + \underbrace{\sum_{|J| > k} \overset{>k}{\lrcorner^{J}}\ {}_{J}\tilde{\mathbb{1}}_{\varnothing}}$$

$$\underset{k = n+1}{\Longrightarrow} \mathbb{1} = \sum_{|J| \leqslant n} \lrcorner^{J}\ {}_{J}\tilde{\mathbb{1}} \Rightarrow \urcorner = \lrcorner^{I}\ {}_{I}\urcorner = \lrcorner^{I}\ \lrcorner^{J}\ {}_{IJ}\tilde{\urcorner} = \lrcorner^{L}\ {}_{L}\tilde{\urcorner}$$

inj

$$\llcorner \overset{-\ \mathbb{N}}{\triangle \mathbb{K}} \boxtimes \mathbb{1} \sqcup \urcorner = \sum_{J} \llcorner^{J}\ \boxtimes_{J} \urcorner \in \ker \varphi \Rightarrow \sum_{J} \lrcorner^{J}\ {}_{J}\tilde{\urcorner} = 0$$

$$\lightning \urcorner \neq 0 \Rightarrow \underset{-}{\deg} \urcorner = k \geqslant 0 \Rightarrow \urcorner = \sum_{|J| \geqslant k} \llcorner^{J}\ {}_{J}\urcorner$$

$$\bigvee_{I} |I| = k$$

$${}_{I}\urcorner \neq 0$$

$$|J| \geqslant k$$

$$J \neq I \Rightarrow \underbrace{N \sqcup I} \cap J \neq \varnothing \Leftarrow \underbrace{N \sqcup I} \cap J = \varnothing \Rightarrow N \sqcup I \subset N \sqcup J \Rightarrow J \subset I \underset{|J| \geqslant k}{\Longrightarrow} J = I$$

$$0 = \lrcorner^{N \sqcup I} \sum_{J} \lrcorner^{J}\ {}_{J}\tilde{\urcorner} = \sum_{J} \lrcorner^{N \sqcup I}\ \lrcorner^{J}\ {}_{J}\tilde{\urcorner} = \lrcorner^{N \sqcup I}\ \lrcorner^{I}\ {}_{I}\tilde{\urcorner} = \pm \lrcorner^{N}\ {}_{I}\tilde{\urcorner} = \pm\ \llcorner^{N}\ \underbrace{\det \lrcorner}_{\text{inv}}\ {}_{I}\tilde{\urcorner} \neq 0 \Rightarrow {}_{I}\tilde{\urcorner} = 0 \Rightarrow {}_{I}\urcorner = 0 \lightning$$