

$$\begin{array}{ccc}
\mathbb{L}_{\triangleleft}^{-\mathbb{N}} \mathbb{1} & \cong & \mathbb{1} = \mathcal{V}^I \overbrace{\mathbb{1} \mathbb{1}} \mathbb{L}_{\triangleleft}^{-\mathbb{N}} \mathbb{1} & \cong & \mathbb{1} = \mathcal{V}^I \overbrace{\mathbb{1} \mathbb{1}} \\
\downarrow \cdot \mathbb{1} & & \downarrow & & \downarrow \\
{}_{2^Q} \mathbb{1} & \cong & {}_I \mathbb{1} \mathbb{1} & \cong & {}_I \mathbb{1} \mathbb{1}
\end{array}$$

$$\mathbb{L}_{\triangleleft}^{-\mathbb{N}} \mathbb{1} = \mathbb{L}_{\triangleleft}^{-\mathbb{N}} \mathbb{K} \mathbb{1}$$

$$\mathcal{V}^1 \dots \mathcal{V}^n \in \mathbb{L}_{\triangleleft}^{-\geq 1} \text{ nilpotent}$$

$$\mathcal{V}^j = \mathbb{L}^I \mathcal{V}^j$$

$$\mathcal{V}^j = \mathbb{L}^I \mathcal{V}^j = \mathbb{L}^i \mathcal{V}^j + \sum_{|I| > 1} \mathbb{L}^I \mathcal{V}^j = \mathbb{L}^i \mathcal{V}^j + \check{\mathcal{V}}^j$$

$$\deg(\mathcal{V}^j - \mathbb{L}^i \mathcal{V}^j) = \deg \check{\mathcal{V}}^j \geq 2$$

$${}_i \mathcal{V}^j \in \mathbb{1}_0$$

$$(1) \det \mathcal{V} \in \mathbb{1}_0^{\mathbb{C}} \text{ inv}$$

$$\mathcal{V} \in {}_p \mathbb{1}_0^{\mathbb{C}} \text{ inv}$$

$$\mathbb{1} := \mathcal{V}^{-1}$$

$$\mathbb{L}^i = (\mathcal{V}^j - \check{\mathcal{V}}^j) {}_j \mathbb{1}^i = \mathcal{V}^j {}_j \mathbb{1}^i - \check{\mathcal{V}}^j {}_j \mathbb{1}^i = \mathcal{V}^j {}_j \mathbb{1}^i + \underbrace{\check{\mathbb{1}}^i}_{\geq 2}$$

$$(2) \mathbb{L}_{\triangleleft}^{-\mathbb{N}} \mathbb{1} = [\mathcal{V}^1 \dots \mathcal{V}^n] \mathbb{1}_{\text{free}} \text{ Algebra } \mathbb{1}$$

$$\mathbb{L}_{\triangleleft}^{-\mathbb{N}} \mathbb{1} \xleftarrow[\text{bij}]{\text{hom } \mathbb{1}} \mathbb{L}_{\triangleleft}^{-\mathbb{N}} \mathbb{1}$$

$$\mathcal{V}^I \mathbb{1} \leftrightarrow \mathbb{L}^I \mathbb{1}$$

$$(3) \mathbb{L}_{\triangleleft}^{-\mathbb{N}} \mathbb{1} = [\mathcal{V}^1 \dots \mathcal{V}^n] \mathbb{1} \text{ Algebra } \mathbb{1}$$

$$\mathbb{L}_{\triangleleft}^{-\mathbb{N}} \mathbb{1} \xleftarrow[\text{surj}]{\text{hom } \mathbb{1}} \mathbb{L}_{\triangleleft}^{-\mathbb{N}} \mathbb{1}$$

$$\mathcal{V}^i \mathcal{V}^n \mathcal{U} \mathcal{L}^i \mathcal{X} \mathcal{I}$$

$$(4) \mathcal{L}_{\mathcal{I}}^{-\geq 1} = \langle \mathcal{V}^1 \dots \mathcal{V}^n \rangle \mathcal{L}_{\mathcal{I}}^{-\mathbb{N}} \text{ ideal}$$

$$(5) \mathcal{L}_{\mathcal{I}}^{-\geq 1} + \underbrace{\mathcal{L}_{\mathcal{I}}^{-\geq 2}} = \langle \mathcal{V}^i + \mathcal{L}_{\mathcal{I}}^{-\geq 2} \rangle \mathcal{I} \text{ Module } \mathcal{I}$$

$$(6) \mathcal{L}_{\mathcal{I}}^{-\geq n} = \langle \mathcal{V}^1 \dots \mathcal{V}^n \rangle \mathcal{L}_{\mathcal{I}}^{-\mathbb{N}} \text{ ideal}$$

$$\mathcal{V}^1 \dots \mathcal{V}^n = \mathcal{L}^1 \dots \mathcal{L}^n \det \mathcal{V}$$

$$\mathcal{V}^N = \mathcal{L}^N \det \mathcal{V}$$

$$\mathcal{V}^1 \dots \mathcal{V}^n = \overbrace{\mathcal{L}_{j_1}^{j_1} \mathcal{V}^1 + \mathcal{V}^{\geq 1}} \dots \overbrace{\mathcal{L}_{j_n}^{j_n} \mathcal{V}^n + \mathcal{V}^{\geq n}} = \overbrace{\mathcal{L}_{j_1}^{j_1} \mathcal{V}^1} \dots \overbrace{\mathcal{L}_{j_n}^{j_n} \mathcal{V}^n}$$

$$+ \underbrace{\overbrace{\mathcal{L}_{j_1}^{j_1} \mathcal{V}^1 \dots \mathcal{V}^{\geq i_1}} \dots \overbrace{\mathcal{L}_{j_n}^{j_n} \mathcal{V}^n}}_{\text{deg} \geq n+1} + \underbrace{\overbrace{\mathcal{L}_{j_1}^{j_1} \mathcal{V}^1 \dots \mathcal{V}^{\geq i_1} \dots \mathcal{V}^{\geq i_2}} \dots \overbrace{\mathcal{L}_{j_n}^{j_n} \mathcal{V}^n}}_{\text{deg} \geq n+2} + \dots + \underbrace{\overbrace{\mathcal{L}_{j_1}^{j_1} \mathcal{V}^1 + \mathcal{V}^{\geq 1}} \dots \overbrace{\mathcal{L}_{j_n}^{j_n} \mathcal{V}^n + \mathcal{V}^{\geq n}}}_{\text{deg} \geq 2n}$$

$$= \overbrace{\mathcal{L}_{j_1}^{j_1} \mathcal{V}^1} \dots \overbrace{\mathcal{L}_{j_n}^{j_n} \mathcal{V}^n} = \mathcal{L}^1 \dots \mathcal{L}^n \det \mathcal{V}$$

$$(1) \Rightarrow (2)$$

surj

$$\text{Ind } \bigwedge_{0 \leq k} \mathfrak{L}^i = \sum_{|J| < k} \mathfrak{V}^J \mathfrak{A}^i + \underbrace{\check{\mathfrak{L}}^i}_{\geq k}$$

$k = 0$ klar

$$0 \leq k \rightsquigarrow j+1: \quad \text{Vor } \mathfrak{L}^i = \sum_{|J| < k} \mathfrak{V}^J \mathfrak{A}^i + \sum_{|J| \geq k} \mathfrak{L}^J \mathfrak{A}^i$$

$$\Rightarrow \mathfrak{L}^i - \sum_{|J| < k} \mathfrak{V}^J \mathfrak{A}^i = \sum_{|J| \geq k} \mathfrak{L}^J \mathfrak{A}^i = \sum_{|J|=k} \mathfrak{L}^J \mathfrak{A}^i + \sum_{|J| > k} \mathfrak{L}^J \mathfrak{A}^i$$

$$\Rightarrow \mathfrak{L}^i - \sum_{|J| < k} \mathfrak{V}^J \mathfrak{A}^i - \sum_{|J| > k} \mathfrak{L}^J \mathfrak{A}^i = \sum_{|J|=k} \mathfrak{L}^J \mathfrak{A}^i = \sum_{|J|=k} \prod_j^J \mathfrak{L}^j \mathfrak{A}^i$$

$$= \sum_{|J|=k} \prod_j^J \underbrace{\mathfrak{V}_{\ell}^j \mathfrak{A}^j + \check{\mathfrak{L}}^j}_{\geq k} \mathfrak{A}^i = \sum_{|J|=k} \underbrace{\mathfrak{V}_{\ell_1}^{\ell_1} \mathfrak{A}^{j_1} + \check{\mathfrak{L}}^{j_1}}_{\geq k-1+2=k+1} \dots \underbrace{\mathfrak{V}_{\ell_k}^{\ell_k} \mathfrak{A}^{j_k} + \check{\mathfrak{L}}^{j_k}}_{\geq k-2+2=k+2} \mathfrak{A}^i$$

$$= \sum_{|J|=k} \left(\mathfrak{V}_{\ell_1}^{\ell_1} \mathfrak{A}^{j_1} \dots \mathfrak{V}_{\ell_k}^{\ell_k} \mathfrak{A}^{j_k} + \underbrace{\mathfrak{V}_{\ell_1}^{\ell_1} \mathfrak{A}^{j_1} \dots \check{\mathfrak{L}}^{i_1} \dots \mathfrak{V}_{\ell_k}^{\ell_k} \mathfrak{A}^{j_k}}_{\geq k-1+2=k+1} + \underbrace{\mathfrak{V}_{\ell_1}^{\ell_1} \mathfrak{A}^{j_1} \dots \check{\mathfrak{L}}^{i_1} \dots \check{\mathfrak{L}}^{i_2} \dots \mathfrak{V}_{\ell_k}^{\ell_k} \mathfrak{A}^{j_k}}_{\geq k-2+2=k+2} + \dots + \underbrace{\check{\mathfrak{L}}^{j_1} \dots \check{\mathfrak{L}}^{j_k}}_{\geq 2k} \right) \mathfrak{A}^i$$

$$= \sum_{|J|=k} \prod_j^J \mathfrak{V}_{\ell}^j \mathfrak{A}^j \mathfrak{A}^i + \underbrace{\mathfrak{L}}_{> k} = \sum_{|L|=k} \mathfrak{V}^L \mathfrak{L}^i + \mathfrak{L}$$

$$\text{inj } \mathfrak{L} \subseteq \mathbb{K} \mathfrak{X} \mathfrak{L} \ni \mathfrak{L} = \sum_I \mathfrak{L}^I \mathfrak{X}_I \mathfrak{L} \in \ker \varphi \Rightarrow \sum_J \mathfrak{V}^J \mathfrak{L} = 0$$

$$\nexists \mathfrak{L} \neq 0 \Rightarrow \deg \mathfrak{L} = k \geq 0 \Rightarrow \mathfrak{L} = \sum_{|J| \geq k} \mathfrak{L}^J \mathfrak{L}$$

$$\bigvee_I \begin{cases} |I| = k \\ \mathfrak{L}^I \neq 0 \\ |J| \geq k \end{cases}$$

$$J \neq I \Rightarrow \underline{NLI} \cap J \neq \emptyset \Leftarrow \underline{NLI} \cap J = \emptyset \Rightarrow NLI \subset NLJ \Rightarrow J \subset I \xrightarrow{|J| \geq k} J = I$$

$$0 = \mathfrak{V}^{N-L} \sum_J \mathfrak{V}^J \mathfrak{L} = \sum_J \mathfrak{V}^{N-L} \mathfrak{V}^J \mathfrak{L} = \mathfrak{V}^{N-L} \mathfrak{V}^I \mathfrak{L} = \pm \mathfrak{V}^N \mathfrak{L} = \pm \mathfrak{L}^N \underbrace{\det \mathfrak{V}}_{\text{inv}} \mathfrak{L} \neq 0 \nexists$$

$$(2) \Rightarrow (3)$$

klar

$$(3) \Rightarrow (4)$$

$$\begin{aligned} \text{surj} \Rightarrow \mathfrak{L}^i &= \mathfrak{V}^J \mathfrak{A}^i = \prod_j \mathfrak{V}^j \mathfrak{A}^i = \mathfrak{A}^i + \sum_{\emptyset \neq J} \underbrace{\prod_j \mathfrak{V}^j \mathfrak{A}^i}_{\geq |J| > 0} \Rightarrow \mathfrak{A}^i = 0 \\ &\Rightarrow \mathfrak{L}^i = \sum_{\emptyset \neq J} \prod_j \mathfrak{V}^j \mathfrak{A}^i \in \langle \mathfrak{V}^1 \dots \mathfrak{V}^n \rangle \mathfrak{L}_{\triangleleft}^{-\mathbb{N}} \\ &\Rightarrow \mathfrak{L}_{\triangleleft}^{-\geq 1} = \langle \mathfrak{V}^1 \dots \mathfrak{V}^n \rangle \mathfrak{L}_{\triangleleft}^{-\mathbb{N}} \subset \langle \mathfrak{V}^1 \dots \mathfrak{V}^n \rangle \mathfrak{L}_{\triangleleft}^{-\mathbb{N}} \end{aligned}$$

$$(4) \Rightarrow (5)$$

$$\begin{aligned} \mathfrak{L}^i &= \mathfrak{V}^j \mathfrak{A}^i \\ \mathfrak{L}_{\triangleleft}^{-\mathbb{N}} \ni \mathfrak{A}^i &= \underbrace{\mathfrak{A}_0^i}_{\in \mathbb{1}} + \underbrace{\mathfrak{A}_{>}^i}_{\geq 1} \\ \Rightarrow \mathfrak{L}^i &= \mathfrak{V}^j \underbrace{\mathfrak{A}_0^i + \mathfrak{A}_{>}^i}_{\geq 2} = \mathfrak{V}^j \mathfrak{A}_0^i + \underbrace{\mathfrak{V}^j \mathfrak{A}_{>}^i}_{\geq 2} \\ \Rightarrow \mathfrak{L}^i + \mathfrak{L}_{\triangleleft}^{-\geq 2} &= \mathfrak{V}^j \mathfrak{A}_0^i + \mathfrak{L}_{\triangleleft}^{-\geq 2} \in \langle \mathfrak{V}^j + \mathfrak{L}_{\triangleleft}^{-\geq 2} \rangle \mathbb{1} \end{aligned}$$

$$(5) \Rightarrow (6)$$

$$\begin{aligned} \mathfrak{L}^i &= \mathfrak{V}^j \mathfrak{L}^i + \tilde{\mathfrak{L}}^i \\ \mathfrak{L}^i &\in \mathbb{I}_0 \\ \tilde{\mathfrak{L}}^i \in \mathbb{L}_{\Delta}^{-\geq 2} &\Rightarrow \mathfrak{L}^1 \dots \mathfrak{L}^n = \det \mathfrak{V} \mathfrak{V}^1 \dots \mathfrak{V}^n \\ \Rightarrow \mathbb{L}_{\Delta}^{-\geq n} = \langle \mathfrak{L}^1 \dots \mathfrak{L}^n \rangle &\mathbb{L}_{\Delta}^{-\mathbb{N}} \subset \langle \mathfrak{V}^1 \dots \mathfrak{V}^n \rangle \mathbb{L}_{\Delta}^{-\mathbb{N}} \end{aligned}$$

$$(6) \Rightarrow (1)$$

$$\begin{aligned} 0 \neq \mathfrak{L}^1 \dots \mathfrak{L}^n = \mathfrak{V}^1 \dots \mathfrak{V}^n \underbrace{\mathbb{I}}_{\in \mathbb{I}_0} &= \underbrace{\mathfrak{L}^1 \dots \mathfrak{L}^n \det \mathfrak{V}}_{\in \mathbb{I}_0} \mathbb{I} = \mathfrak{L}^1 \dots \mathfrak{L}^n \underbrace{\det \mathfrak{V} \mathbb{I}} \\ \Rightarrow \det \mathfrak{V} \mathbb{I} = 1 &\Rightarrow \det \mathfrak{V} \in \mathbb{I}_0^{\text{C inv}} \end{aligned}$$

$$\begin{array}{ccccc} \mathbb{I} & \xleftarrow{\beta} & \mathbb{L}_{\Delta}^{-\mathbb{N}} & \xleftarrow{\sqsupset} & \mathbb{L}_{\Delta}^{-\tilde{\mathbb{I}}} \\ & & \uparrow \mathfrak{U} & & \\ \tilde{\mathbb{I}} & \xrightarrow{\sqsubset} & \mathbb{L}_{\Delta}^{-\tilde{\mathbb{I}}} & & \end{array}$$

$$\gamma \mathbb{I} = \tilde{\mathbb{I}} \in \tilde{\mathbb{I}} \subset \mathbb{L}_{\Delta}^{-\tilde{\mathbb{I}}} \text{ Sub-Alg}$$

$$\tilde{\mathbb{I}}_{\emptyset} = \mathbb{I}$$

$$\det \mathfrak{V} \in \mathbb{I}_0^{\text{C inv}}$$

$$\mathbb{L}_{\Delta}^{-\mathbb{N}} = [\mathfrak{V}^1 \dots \mathfrak{V}^n] \tilde{\mathbb{I}}_{\text{free}} \text{ Algebra } \mathbb{I}$$

$$\mathbb{L}_{\Delta}^{-\mathbb{N}} \xleftarrow[\text{bij}]{\text{hom } \mathbb{I}} \mathbb{L}_{\Delta}^{-\mathbb{N}}$$

$$\mathcal{V}^I \tilde{\mathbb{1}} \leftarrow \mathbb{L}^I \mathbf{x} \mathbb{1}$$

surj

$$\text{Ind } \bigwedge_{0 \leq k} \mathbb{1} = \sum_{|J| < k} \mathcal{V}^J \tilde{\mathbb{1}} + \sum_{|J| \geq k} \mathcal{V}^J \mathbb{1}_J$$

$k = 0$ klar

$$0 \leq k \nrightarrow j + 1: \text{Vor } \mathbb{1} - \sum_{|J| < k} \mathcal{V}^J \tilde{\mathbb{1}} = \sum_{|J| \geq k} \mathcal{V}^J \mathbb{1}_J$$

$$\bigwedge_{|J| \geq k} \bigvee_{\mathbb{1}_J \in \mathbb{1}} \mathbb{1}_J = \tilde{\mathbb{1}}_{\emptyset}$$

$$\Rightarrow \mathbb{1} - \sum_{|J| \leq k} \mathcal{V}^J \tilde{\mathbb{1}} = \mathbb{1} - \sum_{|J| < k} \mathcal{V}^J \tilde{\mathbb{1}} - \sum_{|J| = k} \mathcal{V}^J \tilde{\mathbb{1}}$$

$$= \sum_{|J| \geq k} \mathcal{V}^J \tilde{\mathbb{1}}_{\emptyset} - \sum_{|J| = k} \mathcal{V}^J \tilde{\mathbb{1}}_J = \underbrace{\sum_{|J| = k} \mathcal{V}^J \tilde{\mathbb{1}}_{\emptyset} - \tilde{\mathbb{1}}_J}_{> 1} + \underbrace{\sum_{|J| > k} \mathcal{V}^J \tilde{\mathbb{1}}_{\emptyset}}_{> k}$$

$$k = n+1 \Rightarrow \mathbb{1} = \sum_{|J| \leq n} \mathcal{V}^J \tilde{\mathbb{1}} \Rightarrow \mathbb{1} = \mathcal{V}^I \mathbb{1} = \mathcal{V}^I \mathcal{V}^J \tilde{\mathbb{1}} = \mathcal{V}^L \tilde{\mathbb{1}}$$

inj

$$\mathbb{L} \mathbb{K} \mathbf{x} \mathbb{1} \mathbb{L} \mathbb{1} = \sum_J \mathbb{L}^J \mathbf{x} \mathbb{1}_J \in \ker \varphi \Rightarrow \sum_J \mathcal{V}^J \tilde{\mathbb{1}} = 0$$

$$\nrightarrow \mathbb{1} \neq 0 \Rightarrow \deg \mathbb{1} = k \geq 0 \Rightarrow \mathbb{1} = \sum_{|J| \geq k} \mathbb{L}^J \mathbb{1}_J$$

$$\bigvee_I |I| = k$$

$${}_I \mathbb{1} \neq 0$$

$$|J| \geq k$$

$$J \neq I \Rightarrow \underline{N \setminus I} \cap J \neq \emptyset \leftarrow \underline{N \setminus I} \cap J = \emptyset \Rightarrow N \setminus I \subset N \setminus J \Rightarrow J \subset I \xrightarrow{|J| \geq k} J = I$$

$$0 = \mathcal{V}^{N \setminus I} \sum_J \mathcal{V}^J \tilde{\mathbb{1}} = \sum_J \mathcal{V}^{N \setminus I} \mathcal{V}^J \tilde{\mathbb{1}} = \mathcal{V}^{N \setminus I} \mathcal{V}^I \tilde{\mathbb{1}} = \pm \mathcal{V}^N \tilde{\mathbb{1}} = \pm \mathbb{L}^N \underbrace{\det \mathbb{V}_I}_{\text{inv}} \tilde{\mathbb{1}} \neq 0 \Rightarrow \tilde{\mathbb{1}} = 0 \Rightarrow {}_I \mathbb{1} = 0 \nrightarrow$$