$$\mathbb{1} = \sum_{n}^{\mathbb{N}} \mathbb{1}_n \text{ comm noeth}$$

$$\mathtt{1} = \sum_{n}^{\mathbb{N}} \mathtt{1}_n \text{ fin gen } \mathbb{1} \text{ mod}$$

$$\mathtt{1} \leftarrow \mathbb{1} \mathbin{⧖} \mathtt{1}$$

$$\mathtt{1}_{m+n} \leftarrow \mathbb{1}_m \mathbin{⧖} \mathtt{1}_n$$

$${}^{t}(\mathtt{1}) = \sum_{n} (\mathtt{1}_n)\, t^{n}$$

$$\mathbb{1} = \mathbb{1}_0 \, \mathbb{L}^{\mathbb{N}\,0|m} : \quad \mathbb{L}^i \in \mathbb{1}_{r_i}$$

$$\daleth = \mathbb{1} \, \daleth^{1|n} : \quad \daleth^k \in \daleth_{s_k}$$

$$\daleth_j \ni \urcorner = \daleth^k \; {}_k\urcorner : \quad {}_k\urcorner \in \mathbb{1}_{j - s_k}$$

$${}_k\urcorner = \sum_{r_1\,{}^1\alpha + \cdot\cdot + r_m\,{}^m\alpha = j - s_k} \mathbb{L}^1_{{}^1\alpha} \cdot\cdot \mathbb{L}^m_{{}^m\alpha}$$

$$\text{fin gen } \daleth_j \ulcorner {}^{\mathbb{L}^0} \dot{\mathbb{1}}_j \leftarrow \daleth_j \sqsupset {}^{\mathbb{L}^0} \dot{\mathbb{1}}_j \text{ fin gen}$$

$${}^{\mathbb{L}^0} \dot{\mathbb{1}}_j \leftarrow \underbrace{\mathbb{1}_0 \mathbb{L}^{\mathbb{N}\,1|m}} \boxtimes {}^{\mathbb{L}^0} \dot{\mathbb{1}}_j$$

$$\urcorner \in {}^{\mathbb{L}^0} \dot{\mathbb{1}}_j \Rightarrow \mathbb{L}^0 \, \underbrace{\mathbb{L}^i \urcorner} = \underbrace{\mathbb{L}^0 \mathbb{L}^i} \urcorner = \underbrace{\mathbb{L}^i \mathbb{L}^i} \urcorner = \mathbb{L}^i \, \underbrace{\mathbb{L}^0 \urcorner}_{= 0} = 0$$

$$\underbrace{\daleth_j \ulcorner^{\mathbb{L}^0} \dot{\mathbb{1}}} \leftarrow \underbrace{\mathbb{1}_0 \mathbb{L}^{\mathbb{N}\,1|m}} \boxtimes \underbrace{\daleth_j \ulcorner^{\mathbb{L}^0} \dot{\mathbb{1}}}$$

$$ {}^{t}(\mathtt{1}) \prod_{i}^{1|m} \underbrace{1 - t^{r_i}} \text{ poly} $$

$$ m = 0: \quad \mathbb{1} = \mathbb{1}_0 \Rightarrow \mathtt{1} \text{ fin gen } \mathbb{1}_0 \text{ mod } \underset{\text{fin}}{\Rightarrow} \mathtt{1} = \sum_{n}^{N} \mathtt{1}_n \Rightarrow {}^{t}(\mathtt{1}) = \sum_{n}^{N} (\mathtt{1}_n)\, t^n \text{ poly} $$

$$ 0 \leqslant m \curvearrowright m+1: \quad 0 \leftarrow \mathtt{1}\,{}_{n+r_0}\!\ulcorner\, {}^{\mathbb{1}^0}\dot{\mathtt{1}} \leftarrow \mathtt{1}_{n+r_0} \xleftarrow{\mathbb{1}^0} \mathtt{1}_n \leftarrow {}^{\mathbb{1}^0}\dot{\mathtt{1}}_n \leftarrow 0 $$

$$ \left({}^{\mathbb{1}^0}\dot{\mathtt{1}}_n\right) - (\mathtt{1}_n) + \left(\mathtt{1}_{n+r_0}\right) - \left(\mathtt{1}\,{}_{n+r_0}\!\ulcorner\,{}^{\mathbb{1}^0}\dot{\mathtt{1}}\right) = 0 $$

$$ \sum_{n}^{\mathbb{N}} \left({}^{\mathbb{1}^0}\dot{\mathtt{1}}_n\right) t^{n+r_0} = {}^{t}\left({}^{\mathbb{1}^0}\dot{\mathtt{1}}\right) t^{r_0} $$

$$ \sum_{n}^{\mathbb{N}} (\mathtt{1}_n)\, t^{n+r_0} = {}^{t}(\mathtt{1})\, t^{r_0} $$

$$ \sum_{n}^{\mathbb{N}} \left(\mathtt{1}_{n+r_0}\right) t^{n+r_0} = {}^{t}(\mathtt{1}) - \sum_{m}^{r} (\mathtt{1}_m)\, t^m $$

$$ \sum_{n}^{\mathbb{N}} \left(\mathtt{1}\,{}_{n+r_0}\!\ulcorner\,{}^{\mathbb{1}^0}\dot{\mathtt{1}}\right) t^{n+r_0} = {}^{t}\left(\mathtt{1} \ulcorner {}^{\mathbb{1}^0}\dot{\mathtt{1}}\right) - \sum_{m}^{r_0} \left(\mathtt{1}_m\!\ulcorner^{\mathbb{1}^0}\dot{\mathtt{1}}\right) t^m $$

$$ 0 = {}^{t}\left({}^{\mathbb{1}^0}\dot{\mathtt{1}}\right) t^{r_0} - {}^{t}(\mathtt{1})\, t^{r_0} + {}^{t}(\mathtt{1}) - \sum_{m}^{r_0} (\mathtt{1}_m)\, t^m - {}^{t}\left(\mathtt{1} \ulcorner {}^{\mathbb{1}^0}\dot{\mathtt{1}}\right) + \sum_{m}^{r_0} \left(\mathtt{1}_m\!\ulcorner^{\mathbb{1}^0}\dot{\mathtt{1}}\right) t^m $$

$$ {}^{t}(\mathtt{1}) \underbrace{1 - t^{r_0}} = {}^{t}\left(\mathtt{1} \ulcorner {}^{\mathbb{1}^0}\dot{\mathtt{1}}\right) - {}^{t}\left({}^{\mathbb{1}^0}\dot{\mathtt{1}}\right) t^{r_0} + \sum_{m}^{r_0} \underbrace{(\mathtt{1}_m) - \left(\mathtt{1}_m\!\ulcorner^{\mathbb{1}^0}\dot{\mathtt{1}}\right)} t^m $$

$$ {}^{t}(\mathtt{1}) \prod_{i}^{0|m} \underbrace{1 - t^{r_i}} = {}^{t}(\mathtt{1}) \underbrace{1 - t^{r_0}} \prod_{i}^{1|m} \underbrace{1 - t^{r_i}} $$

$$ = \left({}^{t}\left(\mathtt{1} \ulcorner {}^{\mathbb{1}^0}\dot{\mathtt{1}}\right) - {}^{t}\left({}^{\mathbb{1}^0}\dot{\mathtt{1}}\right) t^{r_0} + \sum_{m}^{r_0} \underbrace{(\mathtt{1}_m) - \left(\mathtt{1}_m\!\ulcorner^{\mathbb{1}^0}\dot{\mathtt{1}}\right)} t^m \right) \prod_{i}^{1|m} \underbrace{1 - t^{r_i}} $$

$$ = \underbrace{{}^{t}\left(\mathtt{1} \ulcorner {}^{\mathbb{1}^0}\dot{\mathtt{1}}\right) \prod_{i}^{1|m} \underbrace{1 - t^{r_i}}}_{\text{poly}} - \underbrace{{}^{t}\left({}^{\mathbb{1}^0}\dot{\mathtt{1}}\right) \prod_{i}^{1|m} \underbrace{1 - t^{r_i}}}_{\text{poly}} t^{r_0} + \prod_{i}^{1|m} \underbrace{1 - t^{r_i}} \sum_{m}^{r_0} \underbrace{(\mathtt{1}_m) - \left(\mathtt{1}_m\!\ulcorner^{\mathbb{1}^0}\dot{\mathtt{1}}\right)} t^m $$

$$r_i = 1 \Longrightarrow \bigvee^{\min}_{k \leqslant m} \bigwedge_{n \geqslant N-1} (\daleth_n) = \frac{n^{k-1}}{(k-1)!} \underbrace{{}^{1}\text{ɿ}}_{\neq 0} + \text{ lower order}$$

$${}^{t}(\daleth)\ \overbrace{1-t}^{m} \text{ poly } \Longrightarrow \bigvee^{\min}_{k \leqslant m} {}^{t}(\daleth)\ \overbrace{1-t}^{k} = {}^{t}\text{ɿ} = \sum_{i}^{N} t^{i}\, {}^{\sharp}_{i}\text{ɿ} \text{ poly : } \quad 0 \neq {}^{1}\text{ɿ} = \sum_{i}^{N} {}^{\sharp}_{i}\text{ɿ}$$

$$\overbrace{1-t}^{-k} = \sum_{j}^{\mathbb{N}} \begin{bmatrix} k+j-1 \\ k-1 \end{bmatrix} t^{j}$$

$${}^{t}(\daleth) = {}^{t}\text{ɿ}\ \overbrace{1-t}^{-k} = \sum_{j}^{\mathbb{N}} \sum_{i}^{N} \begin{bmatrix} k+j-1 \\ k-1 \end{bmatrix} {}^{\sharp}_{i}\text{ɿ}\, t^{i+j} = \sum_{n}^{\mathbb{N}} t^{n} \sum_{N > i \leqslant n} \begin{bmatrix} k+n-i-1 \\ k-1 \end{bmatrix} {}^{\sharp}_{i}\text{ɿ}$$

$$(\daleth_n) = \sum_{N > i \leqslant n} \begin{bmatrix} k+n-i-1 \\ k-1 \end{bmatrix} {}^{\sharp}_{i}\text{ɿ} \underset{n \geqslant \overline{N}-1}{=} \sum_{i}^{N} \begin{bmatrix} k+n-i-1 \\ k-1 \end{bmatrix} {}^{\sharp}_{i}\text{ɿ} = \frac{n^{k-1}}{(k-1)!} \underbrace{{}^{1}\text{ɿ}}_{\neq 0} + \text{ lower order}$$