$$\text{free } \daleth = \underbrace{\llcorner^{0}\cdots\llcorner^{n}}\mathbb{I} \supset \gimel \Rightarrow \gimel = \underbrace{\gimel^{0}\cdots\gimel^{m}}\mathbb{I} \text{ free } m \leqslant n$$

$$\gimel \cap \underbrace{\llcorner^{0}\cdots\llcorner^{k-1}}\mathbb{I} = \underbrace{\gimel^{0}\cdots\gimel^{j}}\mathbb{I} \text{ free } j \leqslant k$$

$$\beth = \frac{a \in \mathbb{I}}{\overbrace{\llcorner^{k} a + \underbrace{\llcorner^{0}\cdots\llcorner^{k-1}}\mathbb{I}} \cap \gimel \neq \varnothing} \lhd \mathbb{I}$$

$$\dot{a} \in \beth \Rightarrow \llcorner^{k}\ \dot{a} + \sum_{i \leqslant k} \llcorner^{i}\ {}_{i}\dot{\daleth} \in \gimel$$

$$\Rightarrow \llcorner^{k}\ \underbrace{a + \acute{a}} + \sum_{i \leqslant k} \llcorner^{i}\ \overbrace{{}_{i}\daleth + {}_{i}\acute{\daleth}} = \overbrace{\llcorner^{k} a + \sum_{i \leqslant k} \llcorner^{i}\, {}_{i}\daleth}^{\in \gimel} + \overbrace{\llcorner^{k} \acute{a} + \sum_{i \leqslant k} \llcorner^{i}\, {}_{i}\acute{\daleth}}^{\in \gimel} \in \gimel \Rightarrow a + \acute{a} \in \beth$$

$$b \in \mathbb{I} \Rightarrow \llcorner^{k}\ ab + \sum_{i \leqslant k} \llcorner^{i}\ {}_{i}\daleth b = \overbrace{\llcorner^{k} a + \sum_{i \leqslant k} \llcorner^{i}\, {}_{i}\daleth}^{\in \gimel} b \in \gimel \Rightarrow ab \in \beth$$

$$\Rightarrow \beth = {}_{k}\gimel^{k}\ \mathbb{I} \Rightarrow \gimel^{k} = \sum_{i \leqslant k} \llcorner^{i}\ {}_{i}\gimel^{k} + \llcorner^{k}\ {}_{k}\gimel^{k} \in \gimel$$

$$\gimel \cap \underbrace{\llcorner^{0}\cdots\llcorner^{k}}\mathbb{I} = \gimel \cap \underbrace{\llcorner^{0}\cdots\llcorner^{k-1}}\mathbb{I} + \gimel^{k}\ \mathbb{I}$$

$$\supset : \quad \llcorner^{k}\ {}_{k}\gimel^{k} + \sum_{i \leqslant k} \llcorner^{i}\ {}_{i}\gimel^{k} \in \gimel \cap \underbrace{\llcorner^{0}\cdots\llcorner^{k}}\mathbb{I}$$

$$\subset : \quad \gimel \cap \underbrace{\llcorner^{0}\cdots\llcorner^{k}}\mathbb{I} \ni \daleth = \sum_{i \leqslant k} \llcorner^{i}\ {}_{i}\daleth = \sum_{i \leqslant k} \llcorner^{i}\ {}_{i}\daleth + \llcorner^{k}\ {}_{k}\daleth \in \gimel \Rightarrow {}_{k}\daleth \in \beth \Rightarrow {}_{k}\daleth = {}_{k}\gimel^{k}\ b$$

$$\Rightarrow \daleth - \gimel^{k}\ b = \sum_{i \leqslant k} \llcorner^{i}\ {}_{i}\daleth + \llcorner^{k}\ {}_{k}\daleth - \llcorner^{k}\ {}_{k}\gimel^{k}\ b - \sum_{i \leqslant k} \llcorner^{i}\ {}_{i}\gimel^{k}\ b$$

$$= \sum_{i \leqslant k} \llcorner^{i}\ \overbrace{{}_{i}\daleth - {}_{i}\gimel^{k} b} + \llcorner^{k}\ \overbrace{{}_{k}\daleth - {}_{k}\gimel^{k} b}^{=0} \in \gimel \cap \underbrace{\llcorner^{0}\cdots\llcorner^{k-1}}\mathbb{I}$$

$$\gimel \cap \underbrace{\llcorner^{0}\cdots\llcorner^{k-1}}\mathbb{I} \cap \gimel^{k}\ \mathbb{I} = \gimel^{k} \begin{cases} b \in \mathbb{I} \\ {}_{k}\gimel^{k} b = 0 \end{cases}$$

$$\gimel^{k}\ b = \llcorner^{k}\ {}_{k}\gimel^{k}\ b + \sum_{i \leqslant k} \llcorner^{i}\ {}_{i}\gimel^{k}\ b \in \gimel \cap \underbrace{\llcorner^{0}\cdots\llcorner^{k-1}}\mathbb{I} \Leftrightarrow {}_{k}\gimel^{k}\ b = 0$$

$$\gimel \cap \underbrace{\llcorner^{0}\cdots\llcorner^{k}}\mathbb{I} = \underbrace{\gimel^{0}\cdots\gimel^{j}\gimel^{k}}\mathbb{I} \text{ free } j + 1 \leqslant k + 1$$