$$\text{IntR } Z \triangleright \mathfrak{p}_1 \cdots \mathfrak{p}_m = \mathfrak{q}_1 \cdots \mathfrak{q}_n \text{ inv}$$

$$\mathfrak{p}_i \underset{\text{prim}}{\triangleleft} Z \underset{\text{prim}}{\triangleright} \mathfrak{q}_j$$

$$\underset{\text{Eind}}{\Longrightarrow} \begin{cases} m = n \geqslant 0 \\ \mathfrak{p}_i = \mathfrak{q}_i \end{cases}$$

$$m = 0: \quad ↯\, n > 0 \Rightarrow Z = \mathfrak{q}_1 \cdots \mathfrak{q}_n \subset \mathfrak{q}_1 \underset{\neq}{\subseteq} Z\, ↯\, n = 0$$

$$1 \leqslant m \curvearrowleft m-1: \text{prim } \mathfrak{p}_1 \supset \mathfrak{p}_1 \cdots \mathfrak{p}_m = \mathfrak{q}_1 \cdots \mathfrak{q}_n \underset{\text{OE}}{\Longrightarrow} \text{ inv } \mathfrak{p}_1 \supset \mathfrak{q}_1 \Rightarrow \mathfrak{p}_1 \prec \mathfrak{q}_1 \Rightarrow \bigvee_{\mathfrak{a} \triangleleft Z} \text{prim } \mathfrak{q}_1 = \mathfrak{p}_1 \mathfrak{a}$$

$$↯\, \mathfrak{q}_1 = \mathfrak{a} \Rightarrow \mathfrak{p}_1 = Z \mathfrak{p}_1 = \underbrace{\mathfrak{q}_1^{-1} \mathfrak{q}_1}\, \mathfrak{p}_1 = \mathfrak{q}_1^{-1} \underbrace{\mathfrak{q}_1 \mathfrak{p}_1} = \mathfrak{q}_1^{-1} \mathfrak{q}_1 = Z\, ↯$$

$$\mathfrak{q}_1 \underset{\neq}{\sqsubseteq} \mathfrak{a} \underset{\text{prim}}{\Longrightarrow} \mathfrak{q}_1 \supset \mathfrak{p}_1 \Rightarrow \mathfrak{q}_1 = \mathfrak{p}_1 \Rightarrow \mathfrak{p}_2 \cdots \mathfrak{p}_m = Z \mathfrak{p}_2 \cdots \mathfrak{p}_m = \mathfrak{p}_1^{-1} \mathfrak{p}_1 \mathfrak{p}_2 \cdots \mathfrak{p}_m = \mathfrak{p}_1^{-1} \mathfrak{q}_1 \cdots \mathfrak{q}_n$$

$$= \mathfrak{q}_1^{-1} \mathfrak{q}_1 \cdots \mathfrak{q}_n = Z \mathfrak{q}_2 \cdots \mathfrak{q}_n = \mathfrak{q}_2 \cdots \mathfrak{q}_n \underset{\text{Ind}}{\Longrightarrow} \begin{cases} m - 1 = n - 1 \\ \bigwedge_{i > 1} \mathfrak{p}_i = \mathfrak{q}_i \end{cases}$$

$$Q \blacktriangleright I = \prod_{\mathfrak{p}} \mathfrak{p}^{I_{\mathfrak{p}}}$$

$$\bigvee_{a \in Z^\times} aI \triangleleft Z \Rightarrow aI = \mathfrak{p}_1 \cdots \mathfrak{p}_m$$

$$aZ \triangleleft Z \Rightarrow aZ = \mathfrak{q}_1 \cdots \mathfrak{q}_n \Rightarrow \bar{a} Z = \overline{\widehat{aZ}} = \bar{\mathfrak{q}}_1 \cdots \bar{\mathfrak{q}}_n$$

$$\Rightarrow I = IZ = \underbrace{Ia}\, \underbrace{\bar{a} Z} = \mathfrak{p}_1 \cdots \mathfrak{p}_m \bar{\mathfrak{q}}_1 \cdots \bar{\mathfrak{q}}_n$$

$$Z \text{ Ded } \Leftrightarrow I \blacktriangleleft Q \Rightarrow I \bar{I} = Z \text{ inv}$$

$$\text{Ded } Z \underset{\text{prim}}{\triangleright} \mathfrak{p} \Rightarrow Z \underset{\max}{\triangleright} \mathfrak{p}$$

$$\mathfrak{p} \underset{\neq}{\subseteq} \mathfrak{a} \underset{\neq}{\triangleleft} Z \overset{\mathfrak{a}}{\underset{\text{inv}}{\Longrightarrow}} \mathfrak{a} \prec \mathfrak{p} \Rightarrow \bigvee_{\mathfrak{b} \triangleleft Z} \mathfrak{p} = \mathfrak{a}\mathfrak{b} \subset Z\mathfrak{b} \subset \mathfrak{b}$$

$$\lightning\ \mathfrak{p} = \mathfrak{b} \overset{\mathfrak{b}}{\underset{\text{inv}}{\Longrightarrow}} Z = \mathfrak{b}\bar{\mathfrak{b}} = \mathfrak{p}\bar{\mathfrak{b}} = \underbrace{\mathfrak{a}\mathfrak{b}}\bar{\mathfrak{b}} = \mathfrak{a}\underbrace{\mathfrak{b}\bar{\mathfrak{b}}} = \mathfrak{a}Z = \mathfrak{a}\ \lightning$$

$$\mathfrak{p} \underset{\neq}{\subseteq} \mathfrak{b} \Rightarrow \mathfrak{p} \text{ not prim}$$

$$\mathfrak{a} \triangleleft Z \text{ DED } \underset{\text{Ex}}{\Longrightarrow} \mathfrak{a} = \mathfrak{p}_1 \cdots \mathfrak{p}_m$$

$$\mathcal{I} = \begin{cases} \mathfrak{a} \underset{\neq}{\triangleleft} Z \\ \mathfrak{a} \neq \mathfrak{p}_1 \cdots \mathfrak{p}_m \end{cases}$$

$$Z \text{ noeth } \Rightarrow \bigvee \mathfrak{a} \underset{\max}{\in} \mathcal{I} \Rightarrow \mathfrak{a} \text{ not prim } \Rightarrow \mathfrak{a} \text{ not max } \Rightarrow \bigvee \mathfrak{a} \underset{\neq}{\subseteq} \mathfrak{b} \underset{\neq}{\triangleleft} Z \overset{\mathfrak{b}}{\underset{\text{inv}}{\Longrightarrow}} \mathfrak{b} \prec \mathfrak{a} \Rightarrow \bigvee_{\mathfrak{b}' \triangleleft Z} \mathfrak{a} = \mathfrak{b}\mathfrak{b}' \subset Z\mathfrak{b}' \subset \mathfrak{b}'$$

$$\lightning\ \mathfrak{b}' = \mathfrak{a} \Rightarrow Z = \mathfrak{a}\bar{\mathfrak{a}} = \mathfrak{b}\mathfrak{b}'\mathfrak{b}'^{-} = \mathfrak{b}\ \lightning$$

$$\mathfrak{a} \underset{\neq}{\subset} \mathfrak{b}' \Rightarrow \dot{\mathfrak{b}} \notin \mathcal{I} \Rightarrow \dot{\mathfrak{b}} = \dot{\mathfrak{p}}_1 \cdots \dot{\mathfrak{p}}_{\dot{m}} \Rightarrow \mathfrak{a} = \mathfrak{b}\mathfrak{b}' = \mathfrak{p}_1 \cdots \mathfrak{p}_m \mathfrak{p}'_1 \cdots \mathfrak{p}'_{m'}$$