$$b \in Q^\times \Longrightarrow Zb \blacktriangleleft Q$$

$$b = p/q{:}p \in Z{:}\quad q \in Z^\times \Longrightarrow q\underbrace{Zb} = Z\underbrace{qb} = Zp \subset Z$$

$$b \in Q^\times \Longrightarrow \ \mathrm{inv}\ \widetilde{Zb}^{-1} = Z\,b^{-1}$$

$$\underbrace{Zb^{-1}}\underbrace{Zb} = Z\underbrace{bb^{-1}} = Z$$