$$
{}^{i|j}_{\ \ k|\ell}\overline{\mathbb{K}}{}^{\pm*} = {}^{i|j}\mathbb{K}^{\mho}_{i|j} \times {}^{k|\ell}\mathbb{K}^{\mho}_{k|\ell} \ \neg\ \begin{cases} {}^{j+k|i+\ell}\mathbb{K}^{\mho}_{j+k|i+\ell} \\[4pt] {}^{i+k|j+\ell}\mathbb{K}^{\mho}_{i+k|j+\ell} \end{cases} = \begin{cases} \dfrac{q\ \in\ {}^{i+k|j+\ell}\mathbb{K}^{\mho}_{i+k|j+\ell}}{jq\overset{*}{j}=q^{-1}} \\[10pt] \dfrac{q\ \in\ {}^{j+k|i+\ell}\mathbb{K}^{\mho}_{j+k|i+\ell}}{jq\overset{*}{j}=q^{-1}} \end{cases}
$$

$$
{}^{p}\overline{\mathbb{K}}{}^{\pm*}_{q} = {}^{p}\mathbb{K}^{\mho}_{p} \times {}^{q}\mathbb{K}^{\mho}_{q} \ \neg\ \begin{cases} {}^{p+q}\mathbb{K}^{\mho}_{p+q} \\[4pt] {}^{p|q}\mathbb{K}^{\mho}_{p|q} \end{cases} = \begin{cases} \dfrac{q\ \in\ {}^{p+q}\mathbb{K}^{\mho}_{p+q}}{jq\overset{*}{j}=q^{-1}} \\[10pt] \dfrac{q\ \in\ {}^{p|q}\mathbb{K}^{\mho}_{p|q}}{jq\overset{*}{j}=q^{-1}} \end{cases}
$$

$$
{}^{n}\overline{\mathbb{K}}{}^{\pm*}_{n} = {}^{n}_{\times}\mathbb{K}^{\mho}_{n} \ \neg\ \begin{cases} {}^{n}_{2}\mathbb{K}^{\mho}_{n} \\[4pt] {}^{n}_{1|1}\mathbb{K}^{\mho}_{n} \end{cases} = \begin{cases} \dfrac{q\ \in\ {}^{n}_{2}\mathbb{K}^{\mho}_{n}}{jq\overset{*}{j}=q^{-1}} \\[10pt] \dfrac{q\ \in\ {}^{n}_{1|1}\mathbb{K}^{\mho}_{n}}{jq\overset{*}{j}=q^{-1}} \end{cases}
$$

$$
{}^{n}\overline{\mathbb{R}}{}^{\pm\imath\mathbb{C}}_{n} = {}^{n}\mathbb{R}^{\mathsf{C}}_{n} \ \neg\ \begin{cases} {}^{n}\mathbb{C}^{\mathsf{C}}_{n} \\[4pt] {}^{n}_{\times}\mathbb{R}^{\mathsf{C}}_{n} \end{cases}
$$

$$
{}^{p|q}\overline{\mathbb{C}}{}^{\pm\imath\mho}_{p|q} = {}^{p|q}\mathbb{C}^{\mho}_{p|q} \ \neg\ \begin{cases} {}^{p|q}_{\times}\mathbb{C}^{\mho}_{p|q} \\[4pt] {}^{p+q}\mathbb{C}^{\mathsf{C}}_{p+q} \end{cases}
$$

$$
{}^{n}\overline{\mathbb{C}}{}^{\pm t\mho}_{n} = {}^{n}\mathbb{C}^{\supset}_{n} \ \neg\ \begin{cases} {}^{n}\mathbb{H}^{\supset}_{n} \\[4pt] {}^{n}_{1|1}\mathbb{R}^{\mho}_{n} \end{cases}
$$

$$
{}^{m}_{2}\overline{\mathbb{C}}{}^{\pm t\mho}_{m} = {}^{m}_{2}\mathbb{C}^{\Omega}_{m} \ \neg\ \begin{cases} {}^{m}_{4}\mathbb{R}^{\Omega}_{m} \\[4pt] {}^{m}_{1|1}\mathbb{H}^{\mho}_{m} \end{cases}
$$

B full trans $\begin{cases} {}^{p}_{2}\overline{\mathbb{R}}{}^{t\mathbb{C}}_{q} = {}^{p}_{2}\mathbb{R}^{\Omega}_{p} \times {}^{q}_{2}\mathbb{R}^{\Omega}_{q} \ \neg\ {}^{p+q}_{2}\mathbb{R}^{\Omega}_{p+q} \\[4pt] {}^{p}_{2}\overline{\mathbb{C}}{}^{t\mathbb{C}}_{q} = {}^{p}_{2}\mathbb{C}^{\Omega}_{p} \times {}^{q}_{2}\mathbb{C}^{\Omega}_{q} \ \neg\ {}^{p+q}_{2}\mathbb{C}^{\Omega}_{p+q} \end{cases}$

D full trans $\begin{cases} {}^{p}\overline{\mathbb{C}}{}^{t\mathbb{C}}_{q} = {}^{p}\mathbb{C}^{\supset}_{p} \times {}^{q}\mathbb{C}^{\supset}_{q} \ \neg\ {}^{p+q}\mathbb{C}^{\supset}_{p+q} \\[4pt] {}^{p}\overline{\mathbb{H}}{}^{t\mathbb{C}}_{q} = {}^{p}\mathbb{H}^{\supset}_{p} \times {}^{q}\mathbb{H}^{\supset}_{q} \ \neg\ {}^{p+q}\mathbb{H}^{\supset}_{p+q} \end{cases}$

$$
{}^{n}\overline{\mathbb{H}}{}^{\pm\imath\mathbb{C}}_{n} = {}^{n}\mathbb{H}^{\mathsf{C}}_{n} \ \neg\ \begin{cases} {}^{n}_{2}\mathbb{C}^{\mathsf{C}}_{n} \\[4pt] {}^{n}_{\times}\mathbb{H}^{\mathsf{C}}_{n} \end{cases}
$$

$$
{}^{p|q}\overline{\mathbb{R}}{}^{\pm J\mho}_{p|q} = {}^{p|q}\mathbb{R}^{\mho}_{p|q} \ \neg\ \begin{cases} {}^{p+q}\mathbb{C}^{\supset}_{p+q} \\[4pt] {}^{p|q}_{\times}\mathbb{R}^{\mho}_{p|q} \end{cases}
$$

$$ {}^{m\pm i\mho}_{2}\mathbb{R}_{m} = {}^{m}_{2}\mathbb{R}^{\Omega}_{m} \neg \begin{cases} {}^{m}_{1|1}\mathbb{C}^{\mho}_{m} \\ {}^{m}_{2}\mathbb{R}^{\mathsf{C}}_{m} \end{cases} $$

$$ {}^{p|q\pm j\mho}_{p|q}\mathbb{H} = {}^{p|q}\mathbb{H}^{\mho}_{p|q} \neg \begin{cases} {}^{p|q}_{\times}\mathbb{H}^{\mho}_{p|q} \\ {}^{p+q}_{2}\mathbb{C}^{\Omega}_{p+q} \end{cases} $$

$$ {}^{n\pm i\mho}_{n}\mathbb{H} = {}^{n}\mathbb{H}^{\supset}_{n} \neg \begin{cases} {}^{n}_{1|1}\mathbb{C}^{\mho}_{n} \\ {}^{n}\mathbb{H}^{\mathsf{C}}_{n} \end{cases} $$

$$ {}^{m\pm j\mho}_{2}\mathbb{R}_{m} = {}^{m}_{2}\mathbb{R}^{\Omega}_{m} \neg \begin{cases} {}^{m}_{\times 2}\mathbb{R}^{\Omega}_{m} \\ {}^{m}_{2}\mathbb{C}^{\Omega}_{m} \end{cases} $$

$$ {}^{p|q\pm i\mho}_{p|q}\mathbb{R} = {}^{p|q}\mathbb{R}^{\mho}_{p|q} \neg \begin{cases} {}^{p|q}\mathbb{C}^{\mho}_{p|q} \\ {}^{p+q}\mathbb{R}^{\mathsf{C}}_{p+q} \end{cases} $$

$$ {}^{p|q\pm *\mathfrak{D}}_{p|q}\mathbb{C} = {}^{p|q}\mathbb{C}^{\mho}_{p|q} \neg \begin{cases} {}^{p|q}\mathbb{H}^{\mho}_{p|q} \\ {}^{p+q}_{2}\mathbb{R}^{\Omega}_{p+q} \end{cases} $$

$$ {}^{n}_{2}\mathbb{C}^{\mathfrak{D}}_{n} = {}^{n}_{2}\mathbb{C}^{\Omega}_{n} \neg {}^{n}_{\times 2}\mathbb{C}^{\Omega}_{n} $$

$$ {}^{n}\mathbb{C}^{\mathfrak{D}}_{n} = {}^{n}\mathbb{C}^{\supset}_{n} \neg {}^{n}\mathbb{C}^{\mathsf{C}}_{n} $$

$$ {}^{p|q\pm *\ni}_{2}\mathbb{C}_{p|q} = {}^{p|q}\mathbb{C}^{\mho}_{p|q} \neg \begin{cases} {}^{p|q}_{2}\mathbb{R}^{\mho}_{p|q} \\ {}^{p+q}\mathbb{H}^{\supset}_{p+q} \end{cases} $$

$$ {}^{n}\mathbb{C}^{\ni}_{n} = {}^{n}\mathbb{C}^{\supset}_{n} \neg {}^{n}_{\times}\mathbb{C}^{\supset}_{n} $$

$$ {}^{m}_{2}\mathbb{C}^{\ni}_{m} = {}^{m}_{2}\mathbb{C}^{\Omega}_{m} \neg {}^{m}_{2}\mathbb{C}^{\mathsf{C}}_{m} $$

$$ {}^{p|q\pm i\mho}_{p|q}\mathbb{H} = {}^{p|q}\mathbb{H}^{\mho}_{p|q} \neg \begin{cases} {}^{p|q}_{2}\mathbb{C}^{\mho}_{p|q} \\ {}^{p+q}\mathbb{H}^{\mathsf{C}}_{p+q} \end{cases} $$

$$ {}^{n\pm j\mho}_{n}\mathbb{H} = {}^{n}\mathbb{H}^{\supset}_{n} \neg \begin{cases} {}^{n}_{\times}\mathbb{H}^{\supset}_{n} \\ {}^{n}_{2}\mathbb{C}^{\supset}_{n} \end{cases} $$

$$ {}^{n}\mathbb{C}^{\mathfrak{C}}_{n} = {}^{n}\mathbb{C}^{\mathsf{C}}_{n} \neg {}^{n}_{\times}\mathbb{C}^{\mathsf{C}}_{n} $$

$$ {}^{n\pm \varkappa\mathfrak{C}}_{n}\mathbb{C} = {}^{n}\mathbb{C}^{\mathsf{C}}_{n} \neg \begin{cases} {}^{n}\mathbb{H}^{\mathsf{C}}_{n} \\ {}^{n}_{2}\mathbb{R}^{\mathsf{C}}_{n} \end{cases} $$

group list

$$^{n}\mathbb{R}_{n}^{\mathsf{C}} \sqsubseteq {}^{n}\mathbb{R}_{n}^{\mathsf{C}} \times {}^{n}\mathbb{R}_{n}^{\mathsf{C}} \sqsubseteq {}^{m+n}\mathbb{R}_{m+n}^{\mathsf{C}}$$

$$^{n}\mathbb{C}_{n}^{\mathsf{C}} \sqsubseteq {}^{n}\mathbb{C}_{n}^{\mathsf{C}} \times {}^{n}\mathbb{C}_{n}^{\mathsf{C}} \sqsubseteq {}^{m+n}\mathbb{C}_{m+n}^{\mathsf{C}}$$

$$^{n}\mathbb{H}_{n}^{\mathsf{C}} \sqsubseteq {}^{n}\mathbb{H}_{n}^{\mathsf{C}} \times {}^{n}\mathbb{H}_{n}^{\mathsf{C}} \sqsubseteq {}^{m+n}\mathbb{H}_{m+n}^{\mathsf{C}}$$

$$^{p:q}\mathbb{R}_{p:q}^{\mathsf{U}} \sqsubseteq {}^{p:q}\mathbb{R}_{p:q}^{\mathsf{U}} \times {}^{p:q}\mathbb{R}_{p:q}^{\mathsf{U}} \sqsubseteq {}^{i:j}\mathbb{R}_{k:l}^{\mathsf{U}}$$

$$^{p:q}\mathbb{C}_{p:q}^{\mathsf{U}} \sqsubseteq {}^{p:q}\mathbb{C}_{p:q}^{\mathsf{U}} \times {}^{p:q}\mathbb{C}_{p:q}^{\mathsf{U}} \sqsubseteq {}^{i:j}\mathbb{C}_{k:l}^{\mathsf{U}}$$

$$^{p:q}\mathbb{H}_{p:q}^{\mathsf{U}} \sqsubseteq {}^{p:q}\mathbb{H}_{p:q}^{\mathsf{U}} \times {}^{p:q}\mathbb{H}_{p:q}^{\mathsf{U}} \sqsubseteq {}^{i:j}\mathbb{H}_{k:l}^{\mathsf{U}}$$

$$^{2n}\mathbb{R}_{2n}^{\Omega} \sqsubseteq {}^{2n}\mathbb{R}_{2n}^{\Omega} \times {}^{2n}\mathbb{R}_{2n}^{\Omega} \sqsubseteq {}^{2m+2n}\mathbb{R}_{2m+2n}^{\Omega}$$

$$^{2n}\mathbb{C}_{2n}^{\Omega} \sqsubseteq {}^{2n}\mathbb{C}_{2n}^{\Omega} \times {}^{2n}\mathbb{C}_{2n}^{\Omega} \sqsubseteq {}^{2m+2n}\mathbb{C}_{2m+2n}^{\Omega}$$

$$^{n}\mathbb{C}_{n}^{\mathsf{D}} \sqsubseteq {}^{n}\mathbb{C}_{n}^{\mathsf{D}} \times {}^{n}\mathbb{C}_{n}^{\mathsf{D}} \sqsubseteq {}^{m+n}\mathbb{C}_{m+n}^{\mathsf{D}}$$

$$^{n}\mathbb{H}_{n}^{\mathsf{D}} \sqsubseteq {}^{n}\mathbb{H}_{n}^{\mathsf{D}} \times {}^{n}\mathbb{H}_{n}^{\mathsf{D}} \sqsubseteq {}^{m+n}\mathbb{H}_{m+n}^{\mathsf{D}}$$

non-group list

$$^{p:q}\mathbb{C}_{p:q}^{\mathsf{U}} \times {}^{p:q}\mathbb{C}_{p:q}^{\mathsf{U}} \qquad \sqsubseteq \qquad {}^{n:n}\mathbb{C}_{n:n}^{\mathsf{U}}$$

$$\mathsf{U} \qquad\qquad\qquad\qquad \mathsf{U}$$

$$^{p:q}\mathbb{C}_{p:q}^{\mathsf{U}} \qquad\qquad \sqsubseteq \qquad\qquad {}^{p+q}\mathbb{C}_{p+q}^{\mathsf{C}}$$

$$^{p:q}\mathbb{C}_{p:q}^{\mathsf{U}} \qquad\qquad \sqsubseteq \qquad\qquad {}^{2n}\mathbb{R}_{2n}^{\Omega}$$

$$\mathsf{U} \qquad\qquad\qquad\qquad \mathsf{U}$$

$$^{p:q}\mathbb{R}_{p:q}^{\mathsf{U}} \qquad\qquad \sqsubseteq \qquad\qquad {}^{p+q}\mathbb{R}_{p+q}^{\mathsf{C}}$$

$$^{2p:2q}\mathbb{C}_{2p:2q}^{\mathsf{U}} \qquad\qquad \sqsubseteq \qquad\qquad {}^{2n}\mathbb{R}_{2n}^{\Omega}$$

$$\mathsf{U} \qquad\qquad\qquad\qquad \mathsf{U}$$

$$^{p:q}\mathbb{H}_{p:q}^{\mathsf{U}} \qquad\qquad \sqsubseteq \qquad\qquad {}^{p+q}\mathbb{H}_{p+q}^{\mathsf{C}}$$

$$^{p:q}\mathbb{C}^{\sqcup}_{p:q} \times {}^{p:q}\mathbb{C}^{\sqcup}_{p:q} \qquad \sqsubseteq \qquad {}^{n:n}\mathbb{C}^{\sqcup}_{n:n}$$

$$\sqcup \qquad\qquad\qquad \sqcup$$

$$^{p:q}\mathbb{R}^{\sqcup}_{p:q} \times {}^{p:q}\mathbb{R}^{\sqcup}_{p:q} \qquad \sqsubseteq \qquad {}^{n:n}\mathbb{R}^{\sqcup}_{n:n}$$

$$^{2p:2q}\mathbb{C}^{\sqcup}_{2p:2q} \times {}^{2p:2q}\mathbb{C}^{\sqcup}_{2p:2q} \qquad \sqsubseteq \qquad {}^{2n:2n}\mathbb{C}^{\sqcup}_{2n:2n}$$

$$\sqcup \qquad\qquad\qquad \sqcup$$

$$^{p:q}\mathbb{H}^{\sqcup}_{p:q} \times {}^{p:q}\mathbb{H}^{\sqcup}_{p:q} \qquad \sqsubseteq \qquad {}^{p+q:p+q}\mathbb{H}^{\sqcup}_{p+q:p+q}$$

$$^{p:q}\mathbb{C}^{\sqcup}_{p:q} \qquad \sqsubseteq \qquad {}^{p+q}\mathbb{H}^{\supset}_{p+q}$$

$$\sqcup \qquad\qquad\qquad \sqcup$$

$$^{p:q}\mathbb{R}^{\sqcup}_{p:q} \qquad \sqsubseteq \qquad {}^{p+q}\mathbb{C}^{\supset}_{p+q}$$

$$^{p:q}\mathbb{C}^{\sqcup}_{p:q} \qquad \sqsubseteq \qquad {}^{p+q}\mathbb{H}^{\supset}_{p+q}$$

$$\sqcup \qquad\qquad\qquad \sqcup$$

$$^{p:q}\mathbb{R}^{\sqcup}_{p:q} \qquad \sqsubseteq \qquad {}^{p+q}\mathbb{C}^{\supset}_{p+q}$$

$$^{2p:2q}\mathbb{C}^{\sqcup}_{2p:2q} \qquad \sqsubseteq \qquad {}^{4p+4q}\mathbb{R}^{\Omega}_{4p+4q}$$

$$\sqcup \qquad\qquad\qquad \sqcup$$

$$^{p:q}\mathbb{H}^{\sqcup}_{p:q} \qquad \sqsubseteq \qquad {}^{2p+2q}\mathbb{C}^{\Omega}_{2p+2q}$$