$$\dashv \in \dashv_{\mathbb{1}}\mathrm{mod}_{\mathbb{1}}$$

$$\vdash \in \dashv^{\sharp} = \mathrm{Hom}\,(\dashv : \mathbb{1})$$

$$\mathbb{1} \xleftarrow[\text{bilin}]{R} \dashv \bowtie \dashv$$

$$\dashv \triangle \mathbb{1}^{\mathbb{N}} \xleftarrow{R_{\vDash}} \dashv \triangle \mathbb{1}^{\mathbb{N}}$$

$$R_{\vDash}\,\underbrace{\dashv^{0} \bowtie \dashv^{1} \bowtie \cdots \bowtie \dashv^{n}} = \sum_{0 \leqslant i < j \leqslant n} (-1)^{j-i}\,\underbrace{\dashv^{i} R \dashv^{j}}\,\dashv^{0} \bowtie \cdots \bowtie \hat{\dashv^{i}} \bowtie \cdots \bowtie \hat{\dashv^{j}} \bowtie \cdots \bowtie \dashv^{n}$$

$$\underbrace{\dashv \bowtie}\,R_{\vDash} - R_{\vDash}\,\underbrace{\dashv \bowtie} = \underbrace{\dashv R}\,\vDash$$

$$R_{\vDash}\,\underbrace{\dashv^{0} \bowtie}\,\underbrace{\dashv^{1} \bowtie \cdots \bowtie \dashv^{n}} = R_{\vDash}\,\underbrace{\dashv^{0} \bowtie \cdots \bowtie \dashv^{n}} = \sum_{0 \leqslant i < j \leqslant n} (-1)^{j-i}\,\underbrace{\dashv^{i} R \dashv^{j}}\,\dashv^{0} \bowtie \cdots \bowtie \hat{\dashv^{i}} \bowtie \cdots \bowtie \hat{\dashv^{j}} \bowtie \cdots \bowtie \dashv^{n}$$

$$= \dashv^{0} \bowtie \sum_{1 \leqslant i < j \leqslant n} (-1)^{j-i}\,\underbrace{\dashv^{i} R \dashv^{j}}\,\dashv^{1} \bowtie \cdots \bowtie \hat{\dashv^{i}} \bowtie \cdots \bowtie \hat{\dashv^{j}} \bowtie \cdots \bowtie \dashv^{n} - \sum_{1 \leqslant j \leqslant n} (-1)^{j-1}\,\underbrace{\dashv^{0} R \dashv^{j}}\,\dashv^{1} \bowtie \cdots \bowtie \hat{\dashv^{j}} \bowtie \cdots \bowtie \dashv^{n}$$

$$= \underbrace{\dashv^{0} \bowtie}\,\underbrace{R_{\vDash} \dashv^{1} \bowtie \cdots \bowtie \dashv^{n}} - \overbrace{\underbrace{\dashv^{0} R}\,\vDash}\,\underbrace{\dashv^{1} \bowtie \cdots \bowtie \dashv^{n}} = \overbrace{\underbrace{\dashv^{0} \bowtie} R_{\vDash} - \underbrace{\dashv^{0} R}\,\vDash}\,\underbrace{\dashv^{1} \bowtie \cdots \bowtie \dashv^{n}}$$

$$\mathbb{1}\ \text{abel} \implies \underbrace{\vdash \vDash}\,R_{\vDash} = R_{\vDash}\,\underbrace{\vdash \vDash}$$

$$\underbrace{\vdash \vDash}\,R_{\vDash}\,\underbrace{\dashv \bowtie} = \underbrace{\vdash \vDash}\,\overbrace{\underbrace{\dashv \bowtie} R_{\vDash} - \underbrace{\dashv R}\,\vDash} = \underbrace{\vdash \vDash}\,\underbrace{\dashv \bowtie}\,R_{\vDash} - \underbrace{\vdash \vDash}\,\overbrace{\underbrace{\dashv R}}\,\vDash$$

$$= \overbrace{\underbrace{\vdash \dashv} I - \underbrace{\dashv \bowtie}\,\underbrace{\vdash \vDash}}\,R_{\vDash} + \overbrace{\underbrace{\dashv R}}\,\vDash \underbrace{\vdash \vDash} \underset{\text{ind}}{=} \underbrace{\vdash \dashv}\,R_{\vDash} - \underbrace{\dashv \bowtie}\,R_{\vDash}\,\underbrace{\vdash \vDash} + \overbrace{\underbrace{\dashv R}}\,\vDash \underbrace{\vdash \vDash}$$

$$= R_{\vDash}\,\underbrace{\vdash \dashv} - R_{\vDash}\,\underbrace{\dashv \bowtie}\,\underbrace{\vdash \vDash} = R_{\vDash}\,\overbrace{\underbrace{\vdash \dashv} I - \underbrace{\dashv \bowtie}\,\underbrace{\vdash \vDash}} = R_{\vDash}\,\underbrace{\vdash \vDash}\,\underbrace{\dashv \bowtie}$$

$$\dashv \triangle \mathbb{1}^{\mathbb{N}} \xleftarrow{\exp R_{\vDash}} \dashv \triangle \mathbb{1}^{\mathbb{N}}$$

$$\mathbb{1} \text{ abel } \Longrightarrow R_{\models} S_{\models} = S_{\models} R_{\models}$$

$$R_{\models} S_{\models} \underbrace{\blacktriangleleft\!\Join} = R_{\models} \underbrace{\blacktriangleleft\!\Join} S_{\models} - R_{\models} \underbrace{\overbrace{\blacktriangleleft S}\,\models} = \underbrace{\blacktriangleleft\!\Join} R_{\models} S_{\models} - \underbrace{\overbrace{\blacktriangleleft R}\,\models} S_{\models} - \underbrace{\overbrace{\blacktriangleleft S}\,\models} R_{\models}$$

$$= \text{ ind } \underbrace{\blacktriangleleft\!\Join} S_{\models} R_{\models} - S_{\models} \underbrace{\overbrace{\blacktriangleleft R}\,\models} - \underbrace{\overbrace{\blacktriangleleft S}\,\models} R_{\models} = S_{\models} \underbrace{\blacktriangleleft\!\Join} R_{\models} - S_{\models} \underbrace{\overbrace{\blacktriangleleft R}\,\models} = S_{\models} R_{\models} \underbrace{\blacktriangleleft\!\Join}$$

$$\mathbb{1} \text{ abel } \Longrightarrow \exp R_{\models} \exp S_{\models} = \exp \overbrace{R+S}_{\models}$$

$$\exp R_{\models} \exp S_{\models} = \exp \overbrace{R_{\models} + S_{\models}} = \exp \overbrace{R+S}_{\models}$$

$$\underbrace{\blacktriangleleft\!\Join} R_{\models}^{n} = R_{\models}^{n} \underbrace{\blacktriangleleft\!\Join} + n \underbrace{\overbrace{\blacktriangleleft R}\,\models} R_{\models}^{n-1}$$

$$n = 1: \quad \text{klar}$$

$$1 \leqslant n \curvearrowright n+1: \quad \underbrace{\blacktriangleleft\!\Join} R_{\models}^{n+1} \underset{\text{ind}}{=} \underbrace{R_{\models}^{n} \underbrace{\blacktriangleleft\!\Join} + n \underbrace{\overbrace{\blacktriangleleft R}\,\models} R_{\models}^{n-1}} R_{\models}$$

$$= R_{\models}^{n} \overbrace{R_{\models} \underbrace{\blacktriangleleft\!\Join} + \underbrace{\overbrace{\blacktriangleleft R}\,\models}} + n \underbrace{\overbrace{\blacktriangleleft R}\,\models} R_{\models}^{n} = R_{\models}^{n+1} \underbrace{\blacktriangleleft\!\Join} + (n+1) \underbrace{\overbrace{\blacktriangleleft R}\,\models} R_{\models}^{n}$$

$$\underbrace{\blacktriangleleft\!\Join} \exp R_{\models} = \exp R_{\models} \underbrace{\blacktriangleleft\!\Join} + \underbrace{\overbrace{\blacktriangleleft R}\,\models} \exp R_{\models}$$

$$\text{LHS } = \underbrace{\blacktriangleleft\!\Join} \underbrace{I + \sum_{1 \leqslant n} \frac{R_{\models}^{n}}{n!}} = \blacktriangleleft\!\Join + \underbrace{\blacktriangleleft\!\Join} \sum_{1 \leqslant n} \frac{R_{\models}^{n}}{n!} = \blacktriangleleft\!\Join + \sum_{1 \leqslant n} \frac{R_{\models}^{n}}{n!} \underbrace{\blacktriangleleft\!\Join} + \underbrace{\overbrace{\blacktriangleleft R}\,\models} \sum_{1 \leqslant n} \frac{n R_{\models}^{n-1}}{n!}$$

$$= \underbrace{I + \sum_{1 \leqslant n} \frac{R_{\models}^{n}}{n!}} \underbrace{\blacktriangleleft\!\Join} + \underbrace{\overbrace{\blacktriangleleft R}\,\models} \exp R_{\models} = \text{ RHS}$$