



$$e^{R_{\neq}} \mathcal{I}_Q \subset \mathcal{I}_{Q+R}$$

$$\mathcal{J} = \frac{1 \in \mathcal{I}_Q}{e^{R_{\neq}} 1 \in \mathcal{I}_{Q+R}}$$

$$1 \in \mathcal{J} \Rightarrow e^{R_{\neq}} \underbrace{1 \mathbf{x} 1}_{\in \mathcal{I}_{Q+R}} = 1 \mathbf{x} \underbrace{e^{R_{\neq}} 1}_{\in \mathcal{I}_{Q+R}} - \underbrace{1 R}_{\in \mathcal{I}_{Q+R}} = \overbrace{e^{R_{\neq}} 1}^{\in \mathcal{I}_{Q+R}} \in \mathcal{I}_{Q+R}$$

$$\Rightarrow \mathcal{J} = \underbrace{1 \triangleleft \mathbb{N}}_{\mathbb{1}} \mathcal{J} \text{ left ideal}$$

$$e^{R_{\neq}} \overbrace{1 \mathbf{x} 1 \mathbf{x} 1} = 1 \mathbf{x} \overbrace{e^{R_{\neq}} 1 \mathbf{x} 1} - \overbrace{1 R} = \overbrace{e^{R_{\neq}} 1 \mathbf{x} 1}$$

$$= 1 \mathbf{x} 1 \mathbf{x} \overbrace{e^{R_{\neq}} 1} - 1 \mathbf{x} \overbrace{1 R} = \overbrace{e^{R_{\neq}} 1} - \overbrace{1 R} = 1 \mathbf{x} \overbrace{e^{R_{\neq}} 1} + \overbrace{1 R} = \overbrace{1 R} = \overbrace{1 R} = \overbrace{e^{R_{\neq}} 1} = \overbrace{1 \mathbf{x} 1 - 1 R 1 \mathbf{x} e^{R_{\neq}} 1}$$

$$\Rightarrow e^{R_{\neq}} \overbrace{1 \mathbf{x} 1 - 1 Q 1 \mathbf{x} 1} = \overbrace{1 \mathbf{x} 1 - 1 Q + R 1} e^{R_{\neq}} 1 \in \mathcal{I}_{Q+R}$$

$$\Rightarrow \mathcal{I}_Q \ni \overbrace{1 \mathbf{x} 1 - 1 Q 1 \mathbf{x} 1} \in \mathcal{J} \Rightarrow \overbrace{1 \mathbf{x} 1 - 1 Q 1 \mathbb{1}} \subset \mathcal{J} \Rightarrow \mathcal{J} = \mathcal{I}_Q$$

$$\begin{array}{ccc} \underbrace{1 \triangleleft \mathbb{N}}_{\mathbb{1}}^{Q+R} & \xleftarrow{e^{R_{\neq}}} & \underbrace{1 \triangleleft \mathbb{N}}_{\mathbb{1}}^Q \\ \uparrow & & \uparrow \\ \underbrace{1 \triangleleft \mathbb{N}}_{\mathbb{1}} & \xleftarrow{e^{R_{\neq}}} & \underbrace{1 \triangleleft \mathbb{N}}_{\mathbb{1}} \end{array}$$

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$$\mathcal{I}_{Q+R} \xleftarrow{e^{R_{\neq}}} \mathcal{I}_Q$$